Possibility measures for valid statistical inference based on censored data\textsuperscript{12}

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\textsuperscript{1}Joint work with my student, Ms. Joyce Cahoon
Typical statistical inference problem: data is *fully observed*.

However, in realistic situations, data may be *censored*.

For example, in clinical trials:
- patient may not show up to a post-treatment appointment
- only a lower bound on patient’s time to remission is available.

This censoring element complicates a statistical analysis.

In particular, *exact* inference is a challenge when nothing about the censoring mechanism is known.
Inferential models (IMs)

- Yesterday I described my IM construction, i.e.,
  - *association* links data, parameter, and auxiliary variables
  - user’s random set predicts unobserved auxiliary variables
  - push random set through the association and compute the corresponding belief/plausibility on parameter space.

- Leads to *provably valid* inference.\(^3\)

- Approach is general, but hard to write down an association without knowing the censoring mechanism.

- However:
  - can write a likelihood without such knowledge
  - then use likelihood to define a *generalized association*\(^4\)(\(^5\))

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\(^3\)M. “False confidence, non-additive beliefs, and valid statistical inference,” from my *BELIEF/SMPS 2018* lecture (arXiv:1607.05051 and *IJAR*)

\(^4\)M. (arXiv:1203.6665, 1511.06733)

\(^5\)Also related to M. (arXiv:1606.02352)
Wanna know more? What if it’s still on sale, 20% off?
Without censoring, we’d get iid samples \((T_1, \ldots, T_n)\) from a distribution depending on a parameter \(\theta\).

Data consists of times/lower bounds and censoring indicators:

\[ Y_i = \left( T_i \wedge C_i, 1_{T_i < C_i} \right), \quad i = 1, \ldots, n. \]

Likelihood doesn’t depend on the censoring mechanism.

Neither does the relative likelihood

\[ (\vartheta, y) \mapsto R_{y, \vartheta} = \frac{L_y(\vartheta)}{L_y(\hat{\theta}_y)}. \]

But its distribution does!
Generalized IM is based on the distribution of $R_Y, \theta$, i.e.,

$$p_y(\{\vartheta\}) = P_{Y|\vartheta, G}\{R_Y, \vartheta \leq R_y, \vartheta\}.$$

Depends on the distribution $G$ of censoring times $C$, which is generally completely unknown.

Paper employs a tweak on Kaplan–Meier to estimate, $G$, the censoring distribution for evaluating $p_y(\cdot)$.

Numerical results suggest that the generalized IM is valid, or at least approximately so.

We should be able to prove this formally...
Log-normal model, 95% intervals for $\psi = \exp\{\mu + \frac{1}{2}\sigma^2\}$

Compare coverage probabilities for
- generalized IM (black)
- asymptotic normality of MLE (red)
- Bayes (green)

Varying $\sigma^2$ and $n = 15, 20, 25, 50$. 

![Graphs showing coverage probabilities for different methods with varying $\sigma^2$.](image-url)
One more commercial

- Peer review + publish-or-perish is detrimental:
  - my livelihood depends on what reviewers think of my work
  - so I have to work on things reviewers will like

- Feedback from peers is important, but needs to be separate from how (junior) researchers are evaluated.

- H. Crane and I have created such a system:

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The end

Thanks!

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