Firearm legislation and Firearm Mortality in the USA: A State Level Study

4-4-2016
Introduction & Problem Statement
The United States is one of many countries that has legislations or laws regulating the manufacture, trade, possession, transfer, record keeping, transport and destruction of firearms, ammunition and firearm accessories. In the United States the right to keep and bear arms is protected by the Second Amendment to the United States Constitution. Many of the federal laws have branched out and have been outlined clearly which protect the rights of the individuals to own firearms. But, within the confined boundaries of the United States, every state has considerably different legislation that could be more or less restrictive than the federal laws pertinent to the possession of guns.

With the recent rise in casualties that involve the use of a firearm, there has been increasing debate on how the laws could be transformed or rewritten in order to retard the violence associated with firearms. The fact that States have a range of different laws protecting the right makes it possible to study the relationship and develop a model between existing gun laws and the associated firearm mortality. The so generated model could be used to emphasize the introduction of certain regulations that could play a positive role in bringing down the firearm mortality. The following study and analysis therefore, is an attempt at understanding the effect of various gun laws on the firearm mortality.

Data and Model Considerations
The data for the analysis was taken from "Firearm Legislation and Firearm Mortality in the USA: a cross-sectional, state level study" by Kalesan et. al. (2016). Additional information regarding the data can be found at:

http://www.cnn.com/2016/03/10/health/gun-laws-background-checks-reduce-deaths/

The data from the above study was compiled to be loaded into R and contains a list of N, X, Y and Z variables. N is a named integer containing the population of the fifty states. X is a 50(states) x 25(laws) matrix containing binary values: 1 if the state had a law in place and 0 if the state did not during the year 2009. Y is a named numeric containing the firearm mortality data in the fifty states for the year 2010. Z is a 50(states) x 5(confounding factors) matrix containing information about the confounding variables unemployment rate, non-firearm homicides, firearm exports, firearm mortality rates in 2009, and firearm ownership rates based on previous studies.

A multivariate Poisson model was used to fit the data with firearm mortality in the year 2010 as the response and with state population as an offset to normalize population sizes. The rate of occurrence or lamda was modelled separately as a linear regression model with the gun law coefficients in X as the predictors and the confounding factors in Z. Equations below show the modelled relationship. A Bayesian approach was employed for modelling the data.

\[ Y_i | \lambda_i \sim \text{Poisson}(N_i \lambda_i) \]  \hspace{1cm} \text{Equation 1}

\[ \log(\lambda_i) = \alpha_0 + \sum_{j=1}^{g} Z_{ij} \alpha_j + \sum_{l=1}^{p} X_{il} \beta_l \]  \hspace{1cm} \text{Equation 2}

\[ \delta = \sum_{i=1}^{50} N_i \lambda_i \]  \hspace{1cm} \text{Equation 3}

Where,
\[ Y_i = \text{Firearm mortality in 2010 in state } i \]
\[ \lambda_i = \text{Rate of firearm mortality (No. of firearm deaths per capita) for state } i \]
\[ N_i = \text{Population of state } i \]
\[ \alpha_0 = \text{Intercept for fitting a linear regression between } \log(\lambda_i) \text{ and the predictors as mentioned in Equation 2} \]
\[ Z_i = \text{Value of confounding factor } j \text{ for } i^{th} \text{ state} \]
\[ \alpha_j = \text{Regression coefficient for the } j^{th} \text{ confounding factor} \]
\[ X_{il} = 1 \text{ if state } i \text{ has the gun law } l \text{ in place, 0 otherwise} \]
\[ \beta_l = \text{Regression coefficient for } X_{il} \]
\[ \delta = \text{Total expected number of firearm deaths in the country} \]

Since, predetermined priors could not be assessed for the parameters in the model, four different models were fit to the data with assumed priors for each model as shown:

- **Model 1** – Using Gaussian uninformative priors for the model coefficients \([\alpha_0, \alpha_j, \beta_l \sim \text{Normal}(0, 100^2)]\).
- **Model 2** – Using Gaussian priors for the model coefficients and a prior on the variance of \(\beta\) \([\alpha_0, \alpha_j \sim \text{Normal}(0, 100^2), \beta_l \sim \text{Normal}(0, \sigma_b^2), \sigma_b^2 \sim \text{InvGamma}(0.01, 0.01)]\).
- **Model 3** – Using Bayesian Lasso model where \([\alpha_0, \alpha_j \sim \text{Normal}(0, 100^2), \beta_l \sim \text{doubleExpo}(0, \sigma_s^2), \sigma_s^2 \sim \text{InvGamma}(0.01, 0.01)]\).
- **Model 4** – Using Bayesian Lasso model where \([\alpha_0 \sim \text{Normal}(0, 100^2), \alpha_j \sim \text{Normal}(0, \sigma_a^2), \sigma_a^2 \sim \text{InvGamma}(0.01, 0.01), \beta_l \sim \text{doubleExpo}(0, \sigma_b^2), \sigma_b^2 \sim \text{InvGamma}(0.01, 0.01)]\).

**Model Selection and Justification for MCMC Algorithm**

All the four models were fit in R using JAGS and MCMC sampling. The first 10000 iterations were discarded as burn in samples and the next 20000 samples were generated for estimating the posterior of the model parameters. For selecting one of the aforementioned four models, convergence criteria was used first to eliminate one of the models and Deviance Information Criteria (DIC) was later used to select the final model for making predictions. Gelman and Rubins shrink factor was plotted for all the parameters determined from the four models. The Gelman and Rubins shrink factor for model 1 indicated lack of convergence for 11 of the parameters. Figure 1 shows the plot for two of the parameters for model 1. Some trace plots showing the generation of the parameter posterior distribution did not resemble white noise process which also confirms the lack of convergence in model 1. Therefore, model 1 was eliminated. Table 1 shows the DIC values for model 2, model 3 and model 4. The DIC values varied very little for the three models but, model 2 was selected to be the final model as the penalized deviance was the least among the three models.

![Figure 1: Shrink Factor Plots for Model 1 Coefficients](image-url)
Diagnostics were performed in order to check if the MCMC algorithm was producing a reliable output for the final model. The Gelman and Rubins shrink factor plots tend to a value of 1 as the number of iterations reach to 30000, which is evidence for the convergence of the model. The trace plots for all the parameters being almost white noise process lead to the conclusion that posterior samples were independent of each other. Autocorrelation (ACF) plots were generated for all the parameters. The ACF decays after a lag of 30 and is insignificant further supporting the assumption of independence. Figure 2 shows sample ACF plots for model 2.

Thus, it can be assumed that the MCMC algorithm produced reliable output for the model parameters.

**Results and Predictions**

The output from the MCMC sampling algorithm for model 2 was analyzed for determining the factors/predictors (gun laws) that were significant in affecting the response (firearm mortality). The 2.5% and 97.5% quantile values for the posterior distribution of all the parameters were referred to determine their statistical significance. If zero was contained between these two quantiles the parameter was considered not significant. As a result, only two of the twenty five gun laws were found to be statistically significant at a 95% confidence level for predicting the firearm mortality. Table 2 below shows the 2.5% and 97.5% quantile values of a few parameters including the two parameters that were statistically significant, in bold.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>2.5% Quant.</th>
<th>97.5% Quant.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta 1</td>
<td>-0.06745</td>
<td>-0.12359</td>
<td>-0.013755</td>
</tr>
<tr>
<td>Beta 10</td>
<td>-0.03081</td>
<td>-0.11547</td>
<td>0.043949</td>
</tr>
<tr>
<td>Beta 23</td>
<td>-0.10450</td>
<td>-0.20594</td>
<td>-0.016650</td>
</tr>
<tr>
<td>Beta 25</td>
<td>0.01790</td>
<td>-0.01941</td>
<td>0.055199</td>
</tr>
</tbody>
</table>

*Table 2: Posterior Distribution Quantiles*

The gun laws corresponding to these two parameters are “Gun dealer license” and “Large magazine ban”. Therefore, using the selected model to fit the observed data, it can be stated that only two gun laws had a significant impact on the reduction of firearm mortality.

Furthermore, predictions for the values of δ were made for various scenarios using the selected model and the estimated parameters. Three scenarios were defined for making the predictions.
Scenario 1: All states remove all gun laws and thus $X_{il} = 0$ for all $i$ and $l$.

Scenario 2: All states keep their current gun laws.

Scenario 3: All states enact all gun laws and thus $X_{il} = 1$ for all $i$ and $l$.

Table 3 shows the estimates for $\delta$ with a 95% confidence interval.

<table>
<thead>
<tr>
<th>$\delta$ (Expected Firearm Deaths)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>33990</td>
<td>31571</td>
<td>24830</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>(32449,35642)</td>
<td>(31224,31921)</td>
<td>(21224,28587)</td>
</tr>
</tbody>
</table>

Table 3: Estimated Firearm Deaths in the USA

Summary of Results
The firearm mortality in the United States was modelled using a Bayesian approach and an MCMC algorithm. Firearm mortality in the states was treated as the response with twenty five varying gun laws as the predictors in presence of five confounding variables. The model used Gaussian prior probabilities and MCMC sampling was employed to generate the posterior distributions for the twenty five parameters. From the parameter estimates, it was concluded that only two of the parameters were significant in reducing the firearm mortality if enacted. In other words, if states enacted the laws that all gun dealers should require a state license and bans be placed on a specific number of rounds, there would be a significant reduction in the firearm mortality. In addition, estimates from the predictions made indicated that if all the states relaxed all the gun laws, then there would be a rise of almost 2420 deaths (8%). On the contrary, if all the states enacted all the twenty five gun laws, the firearm mortality would drop by 6740 (21%).

Limitations and Recommendations
The main limitations of this study and the analysis conducted are that the model is sensitive to the priors that we use. Since in this case, an uninformative prior was used, the model is highly shifted to the observed data and hence the results might differ if an informative prior was used. It is appropriate to conclude if a relationship positive or negative exists between the various gun laws and the firearm mortality, however the numerical estimates for these relationships highly depend on the model chosen for the analysis and the observed data. If this model were to be used for predicting extrapolated estimates for scenarios such as described in the previous section, the estimates can be misleading.

To build on this study, gun law data and firearm mortality data can be collected for multiple years and a model developed accounting for how various gun laws have performed over a period of time. This would normalize the effect of outliers. Additionally, this study only considered five confounding factors that were assumed to be significant in determining a relationship between the response and the predictor. Further research should be conducted to identify additional confounding factors which could be social, economic or religious in nature.
Appendix

```
rm(list=ls())
library(rjags) # Loading rjags package
setwd("C:/Users/Srikanth/Documents/NCSU/ST 590 (Applied Bayesian Analysis)/Take Home Exam 1")
guns = load("guns.RData") # Loading the R data set guns
guns # To view the contents of the R data set guns
Y = Y
N = N
X = X
n = length(Y) # Number of states
p = ncol(Z) # Number of confounding variables
m = ncol(X) # Number of predictors
Xp1 = matrix(0, nrow=50, ncol=25) # Scenario 1 with no gun laws enacted
Xp3 = matrix(1, nrow=50, ncol=25) # Scenario 3 with all gun laws enacted

# Model 1
model_string1 <- "model{

# Likelihood
for(i in 1:n){
  Y[i] ~ dpois(K[i])
  K[i] <- N[i]*lamda[i]
  log(lamda[i]) <- alpha.0 + inprod(Z[i,],alpha[]) + inprod(X[i,],beta[])
}

# Defining Priors
alpha.0 ~ dnorm(0,0.0001)
for(j in 1:p){
  alpha[j] ~ dnorm(0,0.0001)
}
for(k in 1:m){
  beta[k] ~ dnorm(0,0.0001)
}
}
"

model1 = jags.model(textConnection(model_string1),
data = list(Y=Y,Z=Z,X=X,N=N,n=n,p=p,m=m), n.chains=3, quiet=TRUE)
update(model1, 10000, progress.bar="none") # 10000 burn in samples
samp1 = coda.samples(model1,
  variable.names=c("beta","alpha.0","alpha"),
n.iter=20000, progress.bar="none") # Additional 20000 samples for posterior
```
summary(samp1)  # Summary of the statistics
plot(samp1)  # Trace plots and posterior distributions for the parameters
gelman.plot(samp1)  # Gelman Rubins shrinkage factor for the parameters
autocorr.plot(samp1)  # Autocorrelation plots for the parameters

# Model 2
model_string2 <- "model{

# Likelihood
for(i in 1:n){
Y[i] ~ dpois(K[i])
K[i] <- N[i]*lamda[i]
log(lamda[i]) <- alpha.0 + inprod(Z[i,],alpha[]) + inprod(X[i,],beta[])
}

# Defining Priors
alpha.0 ~ dnorm(0,0.0001)

for(j in 1:p){
alpha[j] ~ dnorm(0,0.0001)
}

for(k in 1:m){
beta[k] ~ dnorm(0,inv.var.b)
}

inv.var.b ~ dgamma(0.01,0.01)
}"

model2 = jags.model(textConnection(model_string2),
data = list(Y=Y,Z=Z,X=X,N=N,n=n,p=p,m=m), n.chains=3, quiet=TRUE)

update(model2, 10000, progress.bar="none")
samp2 = coda.samples(model2,
variable.names=c("beta","alpha.0","alpha"),
n.iter=20000, progress.bar="none")
dic2 = dic.samples(model2,
variable.names=c("beta","alpha"),
n.iter=20000, progress.bar="none")  # DIC for model 2

summary(samp2)
plot(samp2)
gelman.plot(samp2)
dic2
autorcorr.plot(samp2)

# Model 3
model_string3 <- "model{
# Likelihood

```r
for(i in 1:n){
  Y[i] ~ dpois(K[i])
  K[i] <- N[i]*lamda[i]
  log(lamda[i]) <- alpha.0 + inprod(Z[i,],alpha[j]) + inprod(X[i,],beta[k])
}
```

# Defining Priors

```r
alpha.0 ~ dnorm(0,0.0001)
for(j in 1:p){
  alpha[j] ~ dnorm(0,0.0001)
}
for(k in 1:m){
  beta[k] ~ ddexp(0,inv.var.b)
}
inv.var.b ~ dgamma(0.01,0.01)
```
# Defining Priors

alpha.0 ~ dnorm(0,0.0001)

for(j in 1:p){
alpha[j] ~ dnorm(0,inv.var.a)
}

for(k in 1:m){
beta[k] ~ ddexp(0,inv.var.b)
}

inv.var.b ~ dgamma(0.01,0.01)
inv.var.a ~ dgamma(0.01,0.01)

}"

model4 = jags.model(textConnection(model_string4),
data = list(Y=Y,Z=Z,X=X,N=N,n=n,p=p,m=m), n.chains=3, quiet=TRUE)

update(model4, 10000, progress.bar="none")

samp4 = coda.samples(model4,
variable.names=c("beta","alpha.0","alpha"),
n.iter=20000, progress.bar="none")
dic4 = dic.samples(model4,
variable.names=c("beta","alpha"),
n.iter=20000, progress.bar="none")

summary(samp4)
plot(samp4)
gelman.plot(samp4)
dic4
autocorr.plot(samp4)

# Predictions using model 2
model_string2p <- "model{

# Likelihood
for(i in 1:n){
Y[i] ~ dpois(K[i])
K[i] ~ N[i]*lamda[i]
log(lamda[i]) <- alpha.0 + inprod(Z[i,],alpha[]) + inprod(X[i,],beta[])
}

# Defining Priors
alpha.0 ~ dnorm(0,0.0001)

for(j in 1:p){
alpha[j] ~ dnorm(0,0.0001)
}"
for(k in 1:m){
  beta[k] ~ dnorm(0,inv.var.b)
}

inv.var.b ~ dgamma(0.01,0.01)

# Predictions for scenario 1
for (i in 1:n){
  Yp1[i] ~ dpois(K1[i])
  K1[i] <- N[i]*lamda1[i]
  log(lamda1[i]) <- alpha.0 + inprod(Z[i,],alpha[]) + inprod(Xp1[i,],beta[])
}
delta1 = inprod(N[,lamda1[]])

# Predictions for scenario 2
for (i in 1:n){
  Yp2[i] ~ dpois(K2[i])
  K2[i] <- N[i]*lamda2[i]
  log(lamda2[i]) <- alpha.0 + inprod(Z[i,],alpha[]) + inprod(X[i,],beta[])
}
delta2 = inprod(N[, lamda2[]])

# Predictions for scenario 3
for (i in 1:n){
  Yp3[i] ~ dpois(K3[i])
  K3[i] <- N[i]*lamda3[i]
  log(lamda3[i]) <- alpha.0 + inprod(Z[i,],alpha[]) + inprod(Xp3[i,],beta[])
}
delta3 = inprod(N[, lamda3[]])

}"}

model2p = jags.model(textConnection(model_string2p),
data = list(Y=Y,Z=Z,X=X,N=N,n=n,p=p,m=m,Xp1=Xp1,Xp3=Xp3), quiet=TRUE)

update(model2p, 10000, progress.bar="none")
samp2p = coda.samples(model2p,
  variable.names=c("Yp1","Yp2","Yp3","delta1","delta2","delta3"),
  n.iter=20000, progress.bar="none")

A = summary(samp2p)
A # Summary of samp2p which includes statewise predictions and nationwise predictions for the
  three scenarios
plot(samp2p)