1 A Primer on Linear Models

1.1 Corrections 23 May 2015

2 Chapter 1 corrections

Fix page: 9 line -7

denominator \((1 - \rho^2)\) is missing, should read

\[
\text{Cov}(e_t, e_s) = \text{Var}(a_t)/(1 - \rho^2)
\]

Fix page: 11 line -5

sign needs changing: \(\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)\)

3 Chapter 2 corrections

Fix page: 15 line -11

missing dot in subscript, end of Example 2.2 should read

any value of \(c\), where \(\overline{y}_i = \sum y_{ij}/n_i\).

Fix page: 23 line 7

matrix in display is \(P_X\), not \(I - P_X\); should read

\[
(I - P_X)z = \frac{1}{10} \begin{bmatrix} 3 & -4 & -1 & 2 \\ -4 & 7 & -2 & -1 \\ -1 & -2 & 7 & -4 \\ 2 & -1 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}
\]

Fix page: 27 line 4

error \(e\) should be \(\hat{e}\) for residuals

orthogonality of the residuals \(\hat{e}\) to the columns of the design matrix \(X\)

Fix page: 31 line 11

dimension of matrix \(A\) should be \(p \times p\)

Fix page: 34 line 7

missing sign

choosing \(u = x + se^{(1)}\) leads to \(Ux = -se^{(1)}\)

4 Chapter 3 corrections

Fix page: 44 line 4

parameter vector \(b\) is missing, should read

\[
X^T X b = \begin{bmatrix} N & n_1 & n_2 & \ldots & n_a \\ n_1 & n_1 & 0 & 0 \\ n_2 & 0 & n_2 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ n_a & 0 & 0 & n_a \end{bmatrix} \begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \ldots \\ \alpha_a \end{bmatrix} = \begin{bmatrix} N \overline{y}_1 \\ n_1 \overline{y}_1 \\ n_2 \overline{y}_2 \\ \ldots \\ n_a \overline{y}_a \end{bmatrix} = X^T y
\]
Fix page: 46 line 2
3, 2 element of $X^T X$ matrix should be $1_b 1_a^T$

Fix page: 46 line -1
$n$ should be $b$
seven of the $a \times b = 4 \times 3 = 12$ cells are observed.

Fix page: 49 line 5
first vector in second display should have first component equal to $-2$, not $-1$, so the vector is $(-2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0)$

5 Chapter 4 corrections

Fix page: 75 line 2
$a$ should be $d$
where $d^T X = \lambda^T$.

Fix page: 75 line 13
linear should be linearly

Fix page: 83 line -1, -7, and -10
missing transposes

\[ \ldots = (y - Xb)^T V^{-1} (y - Xb) \]
\[ = (y - Xb_{GLS})^T V^{-1} (y - Xb_{GLS}) \]

and
\[ \hat{\sigma}^2_{GLS} = \ldots = (y - Xb_{GLS})^T V^{-1} (y - Xb_{GLS}) / (N - r) \]

Fix page: 84 line 17
missing subscript of $\sigma^2$
Then it can be shown that $Var(e_i) = \sigma^2_a / (1 - \rho^2)$ and the covariance

Fix page: 86 line 16
missing $\sigma^2$
$\sigma^2 VX = (\sigma^2 I_N + \tau^2 1_N 1_N^T)$

Fix page: 88 line 3,4
subscript errors

\[ E(\bar{y}_i) = \beta_0 + \beta_1 + \beta_2 n^{-1} \sum_j x_{ij} \]
\[ = \beta_0 + \beta_1 + \beta_2 \bar{x}_i + \beta_2 n^{-1} \sum_j (x_{ij} - \bar{x}_i)^2 \]

Fix page: 91 line -15
Example 4.7 should be Example 4.8

Fix page: 92 line -5
transposes in wrong place, should read
\[ \ldots \text{ if } R \text{ is square and nonsingular and } RVR^T = I, \text{ then } \ldots \]

Fix page: 95 line -7
square in wrong place in Exercise 4.28, should read
\[
\text{Var}(\hat{\beta}_1) = \frac{\sigma^2}{\left(\sum \sum (x_{ij} - \bar{x})^2\right)}
\]

Fix page: 96 line 7,8
missing parentheses in second piece, should read

\[
\text{Var}((\mu + e)^T\mathbf{P}(\mu + e)) = \mathbb{E}[((\mu + e)^T\mathbf{P}(\mu + e)) - (\mathbb{E}[(\mu + e)^T\mathbf{P}(\mu + e)])^2]
\]

Fix page: 107 line both figures
the horizontal axes (u) in both graphs should go to 30, not 3

6 Chapter 5 corrections

Fix page: 115 line 4
In ANOVA table for Example 5.3, the noncentrality parameter for group is missing \(n_i\) in the sum and should read \(\sum_{i=1}^{a} n_i (\alpha_i - \bar{\alpha})^2\)

Fix page: 117 line 15
projection matrix in denominator of (5.11) is incorrect, should read

\[
r^2 = \frac{y^T(Px - P_1)y}{y^T(I - P_1)y}
\]

Fix page: 117 line -8
right hand side of (5.12) should be squared
... and the expression is the squared sample correlation between the response and fitted values

\[
R^2 = \frac{\left(\sum_i (\hat{y}_i - \bar{y})(y_i - \bar{y})\right)^2}{\sum_i (\hat{y}_i - \bar{y})^2 \sum_i (y_i - \bar{y})^2}
\]

Fix page: 123 line 11
index subscript is upper case
Let \(X_N, N = 1, 2, \ldots\) be a sequence of random ...

7 Chapter 6 corrections

Fix page: 125 line -7
Corollary 5.2, not (nonexistent) Corollary 5.5

Fix page: 126 line -2, -3
no need for giant braces; equation (6.4) should read

\[
f(y|b, \sigma^2) = (2\pi)^{-N/2}(\sigma^2)^{-N/2}\exp\left\{\frac{-1}{2\sigma^2}(b^T X^T X b)\right\}\times\exp\{w_1(b, \sigma^2)T_1(y) + w_2(b, \sigma^2)T_2(y)\}.
\]
Fix page: 127 line -11
change sign of second term
\[-(N/2)\frac{1}{\sigma^2} + \frac{1}{2(\sigma^2)^2}Q(\hat{b})\]

Fix page: 133 line 7
missing \(n_i\) in first sum
\[= \sum_{i=2}^{a} n_i(y_{i.} - \bar{y}_i)^2 - \frac{1}{N} \left[ \sum_{i=2}^{a} n_i(y_{i.} - \bar{y}_i) \right]^2\]

Fix page: 142 line -9 and -9
\(\tau\) should be \(m\) in the probability statements
\[Pr(|t_j| < t_{N-r,\alpha/2} \text{ for all } j) = Pr(m \in B) < 1 - \alpha\]
or
\[Pr(|t_j| \geq t_{N-r,\alpha/2} \text{ for at least one } j) = Pr(m \notin B) > \alpha\]

Fix page: 146 line -6
constant and degrees of freedom should both be \(a(n-1)\)
...and independently \(a(n-1)\sigma^2/\sigma^2 \sim \chi^2_{a(n-1)}\)

Fix page: 146 line -3
degrees of freedom in Equation (6.29) should be \(a(n-1)\) to read
\[(\bar{y}_i - \bar{y}_j) - \frac{\hat{\sigma}}{\sqrt{n}} q_{a,a(n-1)}^* \leq \alpha_i - \alpha_j \leq (\bar{y}_i - \bar{y}_j) + \frac{\hat{\sigma}}{\sqrt{n}} q_{a,a(n-1)}^*\]

Fix page: 147 line 6,7
degrees of freedom in Equation (6.30) should be \(a(n-1)\) to read
\[\sum_{i} u_i \bar{y}_i - \frac{\hat{\sigma}}{\sqrt{n}} q_{a,a(n-1)}^* \times \frac{1}{2} \sum_{i} |u_i| \leq \sum_{i} u_i \tau_i \leq \sum_{i} u_i \bar{y}_i + \frac{\hat{\sigma}}{\sqrt{n}} q_{a,a(n-1)}^* \times \frac{1}{2} \sum_{i} |u_i|\]

Fix page: 151 line -4
Prove Corollary 6.1, not 6.2.

Fix page: 151 line -2
add where to evaluate derivative
normal Gauss-Markov model with respect to \(\sigma^2\) evaluated at \(\hat{\sigma}^2 = \hat{\sigma}_{MLE}^2\) is negative:

Fix page: 152 line -8,-7
reword second sentence of Exercise 6.9
Construct the remaining \(a + n - 1\) linearly independent functions of \(\gamma_{ij}\)'s and show how they are confounded with the main effects \(\alpha_i\)'s and \(\beta_j\)'s.

Fix page: 153 line 3
Add instruction to Exercise 6.11
Use \(\sigma^2 = 0.1, 1, 10\).

Fix page: 153 line 9
Correct the Cauchy-Schwarz inequality
\[(u^T w)^2 \leq \|u\|^2 \times \|w\|^2\]
8 Chapter 7 corrections

Fix page: 173 line -4
missing parentheses, wrong projection matrix in denominator

$$F = \frac{y^T(P_Z - P_X)y/(r(Z) - r(X))}{y^T(I - P_Z)y/(N - r(Z))}.$$ 

9 Chapter 8 corrections

Fix page: 181 line -1
drop bar to fix expression for $SSE$

$$SSE = y^T(I - P_x)y = \sum_{i=1}^{n} (y_{ij} - \bar{y}_i)^2$$

Fix page: 183 line 3,4
divide by upper case $N$, not lower case $n$

$$2\phi_a\sigma^2/N = \frac{1}{N} \sum n_i \alpha^2_i - \bar{\alpha}^2 = \alpha^T \frac{1}{N} D\alpha - \frac{1}{N^2} \alpha^T D11^T D\alpha = \alpha^T \left[ \frac{1}{N} D - \frac{1}{N^2} D11^T D \right] \alpha$$

Fix page: 183 line 19
long expression in exponent in equation (8.4)

$$E(E(e_{Us}|\alpha_i')) = (1 - 2s\sigma^2)^{-(a-1)/2} E(e_{Us}^2/(1 - 2s\sigma^2)^{2\lambda_a})$$

Fix page: 188 line -8
$\gamma$ not $\tau$

by writing $\gamma_{ij} = v_{ij} - \bar{v}_i$.

Fix page: 188 line -6
display not broken up

$$\bar{\gamma}_i = \bar{v}_i$$
$$\bar{\gamma}_{ij} - \bar{\gamma}_i = \bar{v}_{ij} - \bar{v}_i$$
$$\gamma_{ij} - \bar{\gamma}_i - \bar{\gamma}_{ij} + \bar{v}_i = v_{ij} - \bar{v}_i - \bar{v}_{ij} + \bar{v}_i$$

Fix page: 189 line 20
subscript upper case $N$

$e \sim N_N(0, R)$

Fix page: 192 line -7
Result A.18(e), not A.7(e)

Fix page: 193 line 14
determinant as written is not correct, should read

$$|X^TV^{-1}X| = \sum_{i=1}^{a} n_i/((\sigma^2 + n_i\sigma_a^2)).$$
Fix page: 200 line 15

correct mean
\[ E \begin{bmatrix} y \\ y_s \end{bmatrix} = X \begin{bmatrix} x_s^T \end{bmatrix} b \]

Fix page: 200 line 23

subscript, not superscript * whose variance is \( \sigma^2(a^T a + 1) = \sigma^2(x_s^T (X^T X)^g x_s + 1) \).

Fix page: 200 line 26

subscript, not superscript *
\[ x_s^T b - y_s \sim N(0, \sigma^2(x_s^T (X^T X)^g x_s + 1)) \]

Fix page: 200 line 27

subscript, not superscript *
\[ Pr\left(-t_{\alpha/2} < \frac{x_s^T b - y_s}{\hat{\sigma}(x_s^T (X^T X)^g x_s + 1)^{1/2}} < t_{\alpha/2}\right) = 1 - \alpha \]

Fix page: 200 line 28,29

subscript, not superscript *
\[ = Pr(x_s^T b - t_{\alpha/2} \hat{\sigma}(x_s^T (X^T X)^g x_s + 1)^{1/2} < y_s < x_s^T b + t_{\alpha/2} \hat{\sigma}(x_s^T (X^T X)^g x_s + 1)^{1/2}) \]

Fix page: 200 line 31 or -3

subscript, not superscript *
\[ x_s^T b \pm t_{\alpha/2} \hat{\sigma}(x_s^T (X^T X)^g x_s + 1)^{1/2}. \]

Fix page: 201 line 1

sign of second term
\[ Var(a^T y - y_s) = a^T \Omega a - 2a^T \omega + \omega_s. \]

Fix page: 201 line 3

missing zero RHS
Unbiasedness again means that \( a^T X b - x_s^T b = 0 \) for all \( b \), or \( X^T a - x_s = 0 \).

Fix page: 201 line 5

change both signs
\[ \Omega a - \omega - X\lambda = 0, \]

Fix page: 204 line -10

Exercise 6.17 should be Exercise 6.25

10 Chapter 9 corrections

Fix page: 210 line -10

second \((I_N - P_X)\) is correct, but superfluous and misleading
\[ \ldots = tr \left[ (I_N - P_X)(X B_{jk} B_{jk}^T X^T + \Sigma_{jk} I_N) \right] = (N - rank(X)) \Sigma_{jk}. \]
11 Appendix A corrections

Fix page: 250 line 15
subscript of identity matrix should be \( n - r \) instead of \( m - r \), should read

\[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} = \begin{bmatrix}
  B_{11}^{-1} b_1 - B_{11}^{-1} B_{12} x_2 \\
  x_2
\end{bmatrix} = \begin{bmatrix}
  B_{11}^{-1} b_1 \\
  0
\end{bmatrix} + \begin{bmatrix}
  -B_{11}^{-1} B_{12} \\
  I_{n-r}
\end{bmatrix} x_2
\]

Fix page: 263 line -6
subscript missing on matrix on left hand side of display, should read

\[
G_2 = C^{-1} \begin{bmatrix}
  I_r & E_1 \\
  E_2 & E_2 E_1
\end{bmatrix} B^{-1}
\]

Fix page: 264 line -5
missing transpose, should read

\[
A^T (AA^T)^{-1} = A^+ 
\]

Fix page: 267 line -8
first matrix is not positive definite, so change the (3, 3) element to 6

\[
\begin{bmatrix}
  1 & -1 & 2 \\
  -1 & 5 & -4 \\
  2 & -4 & 6
\end{bmatrix}
\]

Fix page: 268 line -4
where to evaluate the derivative

\[
\ldots \text{ with respect to } t \text{ at } t = 0 \text{ is equal to } \ldots
\]

Fix page: 268 line -1
incorrect formula for Binomial Inverse Theorem; should read

\[
(A + UBV)^{-1} = A^{-1} - A^{-1}U(B^{-1} + VA^{-1}U)^{-1}VA^{-1}
\]

12 Acknowledgements

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JF Monahan, last update 23 May 2015