1. Assume $Y|\mu, \Sigma$ is multivariate normal with known mean vector $\mu$ and covariance matrix $\Sigma$. Show that the inverse Wishart distribution is the conjugate prior for $\Sigma$.

2. Consider the model $y_i \sim N(0, \sigma^2)$ for $i = 1, \ldots, n$. Consider four estimators of $\sigma^2$:
   
   (a) Classical: $\hat{\sigma}^2 = s^2$, the sample variance
   
   (b) Objective Bayes: the posterior mean of $\sigma^2$ under Jefferys prior for $\sigma^2$
   
   (c) Subjective Bayes 1: the posterior mean of $\sigma^2$ with vague prior $\sigma^2 \sim \text{InvGamma}(0.001, 0.001)$
   
   (d) Subjective Bayes 2: the posterior mean of $\sigma^2$ with tight prior $\sigma^2 \sim \text{InvGamma}(10, 10)$

   Give a closed-form expression for each estimator. Compare these estimators in terms of mean squared error and bias separately for $n = 10$ and 50, and with the true value $\sigma^2 = 0.5, 1, 1.5, \ldots, 5$. Summarize your results in 1-2 plots. Discuss your main conclusions in 1-2 paragraphs.

3. Read
   

   and the corresponding discussion papers (use the link on the “code” page of the course website).

   (a) Identify the three strongest (in your opinion) objectives to Bayesian statistics
   
   (b) Craft a 3-4 sentence rebuttal to each of the three criticisms.