Show all your work (including justifications for steps taken), and clearly state all assumptions made!!

1. (5 points) What constitutes a random sample of size $n$?

2. (5 points) How is the discipline of statistical inference different from the discipline of probability?

3. (10 points)
   
   (a) What is a statistic?

   (b) Let $Y_1, Y_2, \ldots, Y_n$ represent a random sample of size $n$ from a population with unknown mean $\mu$. Is $\frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \mu)^2$ a statistic? How about $\frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \overline{Y})^2$? Make sure you say why or why not.
4. (20 points)

(a) State the Central Limit Theorem and describe its importance.

(b) Illustrate the implications of the Central Limit Theorem on a random sample of size 100 from a Gamma distribution with parameters $\alpha = 10$ and $\beta = 5$. 
5. (15 points) A large industry has an average wage of $5.00 per hour with a standard deviation of $1.00. The industry has 64 workers of a certain ethnic group. These workers have an average wage of $5.25 per hour. Is it reasonable to believe that the ethnic group is a random sample of workers from the industry? (Calculate the probability of obtaining a sample mean greater than or equal to $5.25 per hour.)
6. (25 points) Let $Y_1, Y_2, \ldots, Y_5$ be a random sample of size 5 from a normal population with a mean of 0 and a variance of 1, and let $\overline{Y} = \frac{1}{5} \sum_{i=1}^{5} Y_i$. Let $Y_6$ be another independent observation from the same population.

(a) What is the distribution of $W = \sum_{i=1}^{5} Y_i^2$? Why?
(b) What is the distribution of $U = \sum_{i=1}^{5} (Y_i - \overline{Y})^2$? Why?
(c) What is the distribution of $U + Y_6^2$? Why?
(d) What is the distribution of $\frac{\sqrt{3}n}{\sqrt{W}}$? Why?
(e) What is the distribution of $\frac{2n}{\sqrt{U}}$? Why?

**BONUS 5 points:** What is the distribution of $\frac{2(3Y_6^2 + Y_1^2)}{U}$? Why?