Question 4 is optional, and may thus be left unanswered.
Show all your work (including justifications for steps taken),
and clearly state all assumptions made!!

1. (20 points) Suppose the random variable $Y$ has a beta distribution with parameters
   $\alpha = 2$ and $\beta = 3$. Determine the name and the associated parameter values for the
distribution of $U = 1 - Y$. Note that simply giving the probability density function
(p.d.f.) of $U$ is not sufficient!
2. (25 points) Suppose the random variables $Y_1$ and $Y_2$ are independent, and they are both (individually) distributed as $\chi^2$ with one degree of freedom. Find the probability density function (p.d.f.) of $U = \sqrt{Y_1 + Y_2}$. You may want to use the following outline:

(a) Find the distribution of $Y = Y_1 + Y_2$.
(b) Find the p.d.f. of $U = \sqrt{Y}$.
3. (30 points) Let the random variable $Y$ have probability density function (p.d.f)

$$f_Y(y) = \begin{cases} 
-\frac{2}{3}y & -1 < y < 0 \\
\frac{2}{3}y & 0 \leq y < 2 \\
0 & \text{otherwise.}
\end{cases}$$

The cumulative distribution function (c.d.f) of $U = Y^2$ has the form

$$F_U(u) = \begin{cases} 
0 & u < 0 \\
0 & 0 \leq u < 1 \\
1 & 1 \leq u < 4 \\
1 & 4 \leq u.
\end{cases}$$

Fill in the blanks.
4. (BONUS, 4 points) The joint probability density function of \( Y_1 \) and \( Y_2 \) is

\[
f(y_1, y_2) = \begin{cases} 
\frac{1}{2} & 0 < y_1 < 1, -y_1 < y_2 < y_1 \\
0 & \text{otherwise.}
\end{cases}
\]

What is the probability that the random variable \( Y_2 \) is less than or equal to the random variable \( Y_1 \)?

REMEMBER: You MUST provide justification for your answer!!