1. Concept Review:
   • Interval estimators

2. Exercises

(a) (Revisit) Let $X_1, \ldots, X_n$ be a random sample from $U(\theta, 1)$
   i. Show that $\frac{X_{(1)}}{1-\theta}$ is a pivot quantity
   ii. Derive a $(1 - \alpha)100\%$ confidence interval for $\theta$ by using the pivotal quantity
   iii. Derive a $(1 - \alpha)100\%$ confidence interval for $\theta$ by pivoting the CDF of $X_{(1)}$

(b) Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Beta}(\theta, 1)$
   i. Derive the LRT for test of $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$
   ii. Invert the LRT in (i.) to construct an one-sided $(1 - \alpha)100\%$ confidence interval for $\theta$.
   iii. Use the fact that $X_1^\theta$ is a pivot quantity. Derive the distribution of $X_1^\theta$ and construct an one-sided $(1 - \alpha)100\%$ confidence interval for $\theta$. Then comment on these two intervals.

(c) Let $X$ be a single observation from exponential$(\theta)$, with mean $\theta$. Show that both of the following methods result in the same interval estimator
   i. Use the fact that $\frac{2X}{\theta}$ is a pivotal quantity and construct an equal-tailed confidence accordingly.
   ii. Construct another equal-tailed confidence interval by pivoting the CDF of $X$.

3. Open for questions.