1. Question 1: What determines the distribution uniquely?

2. Question 2: When can you change the differentiation and integration?

3. Suppose that four couples go to a concert where there are exactly eight seats in a row and seats are allocated randomly to each individual. What is the probability that at least one couple sit together.

4. Two couples and one single person are seated at random in a row of five chairs. What is the probability that neither of the couples sits together in adjacent chairs?

5. In how many ways can 4 different pair of shoes be laid in a row if
   (i) there are no restrictions on the laying arrangement.
   (ii) all the four left-foot shoes are laid together and all the four right-foot shoes are also laid together.
   (iii) at least one pair of shoes are not laid next to each other.

6. Let $X$ be a discrete random variable taking values 0,1,2,... with probabilities
   $P(X = 0) = \frac{1}{2}(1 + e^{-2})$ and for $k=1,2,...$, $P(X = k) = \frac{2^{k-1}}{k!} e^{-2}$.
   Find the moment generating function of $X$.
   Hence or otherwise calculate the coefficient of skewness $\alpha_3 = \mu_3/\mu_2^{3/2}$, where $\mu_2$ and $\mu_3$ are respectively the second and third central moments of $X$.

7. Assume that the moment-generating function of a random variable $X$ is
   $$ M_X(t) = \left(\frac{3}{5} + \frac{2}{5}e^t\right)^6 $$
   (i) What is the distribution of $X$?
   (ii) Compute the $E(X)$ and $Var(X)$.

8. Let $X$ be a discrete random variable taking values 0,1,2,... with probabilities
   $P(X = 0) = 1 - \alpha + \alpha p$ and for $k=1,2,...$, $P(X = k) = \alpha pq^k$, where $q=1-p$, $0 < p < 1$, $0 < \alpha < 1$. (i) Find the moment generating function of $X$. (ii) Compute the $E(X)$ and $Var(X)$ based on the moment generating function obtained from (i).

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9. Let $X_n$, $n=1,2,...$, be a sequence of Poisson random variables, and the expected value of $X_n$ is $n\lambda$, where $\lambda > 0$ is a constant. Define $Y_n = \sqrt{n}(X_n/n - \lambda)$ for $n=1,2,...$.

(a) What is the moment generating function of $X_n$?
(b) Compute the moment generating function of $Y_n$.
(c) Show that the sequence $Y_n$ converge in distribution to a Normal random variable with mean 0 and variance $\lambda$, as $n \to \infty$.
(d) Computing $\lim_{n \to \infty} P(X_n \leq n\lambda)$. 
