# 1.7
(a) 
\[ P(\text{scoring } i \text{ points}) = \begin{cases} 
1 - \frac{1}{A^2} & \text{if } i = 0 \\
\frac{1}{A^2} \left[ \frac{(6-i)^2}{5^2} - \frac{(5-i)^2}{5^2} \right] & \text{if } i = 1, \ldots, 5 
\end{cases} \]

(b) 
\[ P(\text{scoring } i \text{ points} \mid \text{board is hit}) = \frac{P(\text{scoring } i \text{ points} \cap \text{board is hit})}{P(\text{board is hit})} \]
\[ P(\text{board is hit}) = \frac{\pi_i}{A} \]
\[ P(\text{scoring } i \text{ points} \cap \text{board is hit}) = \frac{\pi_i}{A} \left[ \frac{(6-i)^2}{5^2} - \frac{(5-i)^2}{5^2} \right] \]

Thus,
\[ P(\text{scoring } i \text{ points} \mid \text{board is hit}) = \frac{(6-i)^2 - (5-i)^2}{5^2} \quad i = 1, \ldots, 5. \]

Which is exactly the distribution of Example 1.2.7.

# 1.24
(a) 
\[ P(\text{A wins}) = \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + \cdots \]
\[ = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n+1} = \frac{\frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} \]
\[ = \frac{3}{5} \]

(b) 
\[ P(\text{A wins}) = \left(\frac{1}{2}\right) + \left(1 - \frac{1}{2}\right)^2 \left(\frac{1}{2}\right) + \left(1 - \frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + \cdots \]
\[ = \frac{\frac{1}{2}}{1 - \left(1 - \frac{1}{2}\right)^2} = \frac{1}{3} \left(\frac{1}{2}\right)^2 \]

Outcome (Suppose A tosses first):
- H or T
- TTH or D
- ABA
- TTTTH etc.
(c) \[ P(\text{A wins}) = \frac{1}{1-(1-p)^2} \]
\[ = \frac{1}{2p-p^2} = \frac{1}{2-p} \]
for all \( 0 < p < 1, 1 < 2-p < 2 \). Thus \( \frac{1}{2-p} > \frac{1}{2} \).

i.e. \( P(\text{A wins}) > \frac{1}{2} \).

OR You can use L'Hôpital's rule to find \( \lim_{p \to 0} \frac{p}{1-(1-p)^2} = \frac{1}{2} \)
and \( \frac{\text{d}}{\text{d}p} \left( \frac{p}{1-(1-p)^2} \right) > 0 \), the prob is increasing in \( p \), and minimum is at zero.

#134 (a) \[ P(\text{Brown Hair}) = P(\text{Brown Hair} | \text{Litter 1}) \cdot P(\text{Litter 1}) + P(\text{Brown Hair} | \text{Litter 2}) \cdot P(\text{Litter 2}) \]
\[ = \left( \frac{2}{5} \right) \left( \frac{1}{2} \right) + \left( \frac{3}{5} \right) \left( \frac{1}{2} \right) = \frac{19}{30} \]

(b) \[ P(\text{Litter 1} | \text{Brown Hair}) = \frac{P(\text{BH} | \text{L1}) \cdot P(\text{L1})}{P(BH)} = \frac{\left( \frac{2}{5} \right) \left( \frac{1}{2} \right)}{\frac{19}{30}} = \frac{10}{19} \]

#136 Let \( X \) = times that target is hit.
\[ P(X=2) = 1 - P(X=0) - P(X=1) \]
\[ = 1 - \left( \frac{4}{5} \right)^0 - 10 \cdot \frac{4}{5} \cdot \left( \frac{1}{5} \right) \]
(to possibilities of which shot hit target)
\[ P(X=2 \mid X=1) = \frac{P(X=2 \cap X=1)}{P(X=1)} = \frac{P(X=2)}{P(X=1)} \]

\[
= \frac{1 - \left(\frac{4}{5}\right)^0 - 10 \cdot \left(\frac{4}{5}\right)^9 \cdot \left(\frac{1}{5}\right)}{1 - \left(\frac{4}{5}\right)^0}
\]

138. (a) \[ P(A) = P(A \cap B) + P(A \cap B^c) \]

But \( A \cap B^c \subset B^c \) and \( P(B^c) = 1 - P(B) = 0 \)

So \( P(A \cap B^c) = 0 \)

Thus, \( P(A) = P(A \cap B) \)

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} = P(A) \]

(b) \( A \subset B \Rightarrow A \cap B = A \)

\[ P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1 \]

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \]

(c) \( A \& B \) are mutually exclusive

\[ P(A \cup B) = P(A) + P(B) \text{ and } A \cap (A \cup B) = A \]

\[ P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)} \]

(d) \[ P(A \cap B \cap C) = P(A \cap (B \cap C)) \]

\[ = P(A \mid B \cap C) \cdot P(B \cap C) \]

\[ = P(A \mid B \cap C) \cdot P(B \mid C) \cdot P(C) \]
1.44. \( P ( \text{at least 10 correct by guessing}) = \sum_{k=10}^{20} \binom{20}{k} \left(\frac{1}{4}\right)^k \cdot \left(\frac{3}{4}\right)^{20-k} = 0.01386 \)

E1. (a) \( S = \{1, k+1\} \)

(b) \( P (\text{only 1 test is needed}) \)
\[
P = P (\text{all } k \text{ test samples are pure})
= P^k.
\]

(b) \( P (\text{k+1 tests are needed}) \)
\[
P = 1 - P (\text{only 1 test is needed})
= 1 - P^k.
\]

E2. \( P (\text{split along gender lines position is 5-3}) \)
\[
= \frac{(\frac{1}{2})^5 \cdot (\frac{1}{2})^3}{(\frac{1}{3})^5 \cdot (\frac{1}{3})^3} = \frac{1}{56} = 0.0179
\]

E3. Consider three adjacent cars as one car, so:

\[\begin{array}{cccc}
\ & \ & \ & \\
\ & \ & \ & \\
\ & \ & \ & \\
\ & \ & \ & \\
\end{array}\]

7 spots in total. 7! choices.

For example, 3 cars are here.
In this spot, 3 cars have 2! possibility of the position.

\( \Rightarrow \) total possibilities for 3 cars together = 3! 7!

\[\text{:. prob.} = \frac{3! \cdot 7!}{9!}\]