Galton's Data  

P=Parents' average ht.  S=Son's ht.

\[
\begin{pmatrix}
P \\
S
\end{pmatrix} \sim N\left( \begin{pmatrix}
68.3 \\
68.1
\end{pmatrix}, \begin{pmatrix}
3.19 & 2.06 \\
2.06 & 6.34
\end{pmatrix}\right)
\]

1. What is the distribution of (P-S)?
   Type of distribution
   Mean _____   Variance ______

P-S is normally distributed with mean 0.2 and variance

\[
(1 -1) \begin{pmatrix}
3.19 & 2.06 \\
2.06 & 6.34
\end{pmatrix} \begin{pmatrix}
1 \\
-1
\end{pmatrix} = 9.53-4.12 = 5.41
\]

2. What proportion of sons' heights exceed their parents' average height by 1"?  
   \[Pr\{S-P>1\} = Pr\{Z>\frac{-1-0.2}{\sqrt{5.41}}\} = 0.3037\]

2. \[Pr\{P-S<-1\} = Pr\{Z< \frac{-1-0.2}{\sqrt{5.41}}\} = 0.3037\]

3. Challenge: Why do you think the S variance is twice that of P? Any theory?

3. Suppose Variance{dad} = Variance{mom} = Variance{son} and mom, dad independent.

\[
\begin{pmatrix}
M \\
D
\end{pmatrix} \sim N\left( \begin{pmatrix}
* \\
*
\end{pmatrix}, \begin{pmatrix}
6.34 & 0 \\
0 & 6.34
\end{pmatrix}\right)
\]

\[
P = (0.5 \ 0.5) \begin{pmatrix}
M \\
D
\end{pmatrix}
\sim N\left( *, (0.5 \ 0.5)\begin{pmatrix}
6.34 & 0 \\
0 & 6.34
\end{pmatrix}(0.5)\right)
\]
$$N( *, 2(\frac{6.34}{4}) )$$

4. For later: what is the correlation between $P$ and $S$?

$$\frac{2.06}{\sqrt{(6.34)(3.19)}}$$