The lecture notes for this particular day are centered on sections 2.4-2.5 in the Devore book. Basically the idea of conditional probability is presented here. Conditional probability states that given an event (event B), it will only occur if an event (event A) has already occurred. The probability statement can be written as follows:

$$P(B|A)$$

This only applies when the events are independent of each other meaning event A has no effect on the probability of event B happening. The other case involves these two events when they are independent. This scenario produces an intersection of the two events (the probability that both events occur). This can be written as follows:

$$P(A \text{ and } B) = P(A)P(B|A) \text{ or } P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

**Example: (Deck of Cards)**

- B = Red Card
- A(1) = Card is a Diamond
- A(2) = Card is a Jack

Pr(A1) = .25 (Probability the card is a diamond)

Pr(A1 | B) = .50 (Probability that the card is a Diamond given it is Red)

Pr(A2 | B) = 1/13 (Probability the card is a Jack of Diamond given it is Red)
This brings us to the subject of mutually exclusive events. The definition of being mutually exclusive (disjoint) means that it is impossible for two events to occur together. Given two events, A and B, they are mutually exclusive if \((A \cap B) = 0\). If these two events are mutually exclusive, they cannot be independent.

**Question:** If A, B are mutually exclusive, then A, B are independent...?

a) Sometimes

b) Always

c) Never

Correct answer is c.

**Example (Mutually Exclusive Events):**

A recent study was conducted using both male and female subjects ages 20-30 that wanted to find the average salary of men vs women.

For the example here, the mutually exclusive events are the subject in the study could not be both female and male at the same time. The subject also cannot be aged 22 or 28 (random selection, any of the ages would work in the range given) at the same time.
This lecture summary covers parts of conditional probability. We went through several examples of how to determine the different probabilities using a Venn diagram. We covered also the difference between independent and dependent events.

Conditional Probability – the probability that one event will occur given that another event has already occurred. 

Notation: Probability that A will occur given B has occurred already = $\Pr(A|B)$.

$$
\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}
$$

Events are independent if $\Pr(A|B) = \Pr(A)$ and are dependent other wise.

Example. Given a Normal Deck of Playing Cards. 

A=Diamond Card   B=Red Card   C=Jack

Example 1) 

$\Pr(A|B) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{2} = 50\%$

$= \textit{Probability of the card being a diamond after knowing that it is a red card}$

$\Pr(A) = \frac{13}{52} = \frac{1}{4} = 25\% = \textit{Probability of the card being a diamond card}$

$\rightarrow \Pr(A|B) \neq \Pr(A)$ which means that events A and B are dependent on each other.

Example 2) Using same event designations as example 1.

$\Pr(C|B) = \frac{2}{52} \cdot \frac{1}{2} = \frac{1}{13} = \textit{Probability of the card being a Jack after knowing that it is a red card}$

$\Pr(C) = \frac{4}{52} = \frac{1}{13} = \textit{Probability of the card being a Jack}$

$\rightarrow \Pr(C|B) = \Pr(C)$ which means that events B and C are independent on each other.

If A is independent of B, then $A'$ is also independent of B.
Event Independence Vs. Mutually exclusive

If events A and B are mutually exclusive of each other then the events will never be independent of each other.

$$\Pr(A|B) = \frac{\Pr(A \land B)}{\Pr(B)} = 0$$

$$\Pr(B) = 0$$

$$\therefore \text{they will never be independent of each other.}$$

If the events are independent of each other then the Venn diagram can be simplified.

\[
\begin{align*}
\Pr(A) + \Pr(A') & = 1 \\
\Pr(B) \times \Pr(A) & = \Pr(A \land B) \\
\Pr(A \land B) + \Pr(A' \land B) & = \Pr(B)
\end{align*}
\]
Homework-0  
Lecture Notes: ST-371-002  
(02/03/2011)

I.  Review from Tues. Feb. 1st.  

a) Conditional Probability: What is the probability event A will happen, given that event B already happened.

\[
Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}
\]

II.  New Material.  

a) Definition: Two events, A and B, are Independent if \(Pr(A|B) = Pr(A)\), and are dependent otherwise.

i.  Example 1: Given a deck of cards, event B=Red Card and event A1=Diamond Card. Are these two events independent?

\[
\cdot Pr(A_1) = 0.25  
\cdot Pr(A_1|B) = 0.50  
\]

**ANSWER:** The two events are dependent.

ii. EXAMPLE 2: In addition to the above statements, event A2=Jack Card. Are events A2 and B independent?

\[
\cdot Pr(A_2|B) = \frac{1}{13}  
\cdot Pr(A_2) = \frac{1}{13}  
\]

**ANSWER:** The two events are independent.  
*Note:* A2 and B’ are also independent.
b) If two events, A and B, are mutually exclusive and \( A \neq 0 \) and \( B \neq 0 \), are A and B independent?

\[
\cdot \Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0}{\Pr(B)} = 0
\]

\[
\cdot \Pr(A) \neq 0
\]

**ANSWER:** The two events are always dependent.

c) If two events, A and B, are independent then the probability tree can be redrawn as shown below:

\[
\cdot \Pr(A \cap B) = \Pr(B) \cdot \Pr(A|B) = \Pr(B) \cdot \Pr(A)
\]

\[
\cdot \text{Also, } \Pr(A \cap B) + \Pr(B \cap A') = \Pr(B)
\]

d) The Following is an example of transferring data from a table to a tree:
Chapter 2.4 Conditional Probability

Notation for this can be written $P(A|B)$, which represents the conditional probability of $A$ given that the event $B$ has occurred.

Definition: For any two events $A$ and $B$ with $P(B) > 0$ the conditional probability of $A$ given that $B$ has occurred is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The Multiplication Rule for $P(A \cap B)$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

- Often used with a tree diagram:

```
          P(B1A) --> P(B1A) \cdot P(A) = P(B \cap A)
          \      \        \         \           \     
          \      \        \         \           \     
          P(A)  P(B1A')   \        \         \           \  
          \      \        \         \           \      \ 
          P(A') P(B1A')   \        \         \           \      \ 
          \      \        \         \           \      \  
          P(B1A)  P(B1A')  \        \         \           \    
```

Bayes' Theorem

Let $A_1, A_2, \ldots, A_k$ be a collection of $k$ mutually exclusive events with prior probabilities $P(A_i)$, $i = 1, \ldots, k$. Then for any other event $B$ for which $P(B) > 0$, the probability of $A_i$ given that $B$ has occurred is
\[ P(A_i | B) = \frac{P(A_i \cap B)}{P(B)} = \frac{\sum_{i=1}^{k} P(B | A_i) P(A_i)}{P(B)} \]

- The transition from the 2nd to 3rd expression results by using the multiplication rule in the numerator and the law of total probability in the denominator.

The Law of Total Probability

Let \( A_1, \ldots, A_k \) be mutually exclusive and exhaustive events. Then for any other event \( B \),

\[ P(B) = P(B | A_1) P(A_1) + \ldots + P(B | A_k) P(A_k) \]

\[ = \sum_{i=1}^{k} P(B | A_i) P(A_i) \]

Partition of \( B \) by mutually exclusive and exhaustive \( A_i \)'s
Chapter 2.5 - Independence

Definition: Two events $A$ and $B$ are independent if
$P(A|B) = P(A)$ and are dependent otherwise.

The Multiplication Rule for $P(AB)$

$A$ and $B$ are independent if and only if
$P(AB) = P(A) \cdot P(B)$

Independence of More Than Two Events

Def: Events $A_1, \ldots, A_n$ are mutually independent
if for every $k \ (k=2,3,\ldots,n)$ and every subset
of indices $i_1, i_2, \ldots, i_k$,

$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdot \ldots \cdot P(A_{i_k})$.

Consider

\[ \begin{align*}
\text{a)} & & \text{series parallel} \\
\text{b)} & & \text{total-cross tied}
\end{align*} \]

\[ \begin{align*}
& a) \quad P[(A_1 \land A_2 \land A_3) \lor (A_4 \land A_5 \land A_6)] \\
& \quad \text{at .9 prob each} \\
& \quad \Rightarrow 1 - [(1-.9)^3]^2 = .927 \\
& b) \quad [1 - (1-.9)^2]^3 = .94
\end{align*} \]
LECTURE 2/3/11 SUMMARY

This lecture first dealt with conditional probability. Conditional probability is the probability of an event given that another event already occurred.

Ex) Probability of A given B

\[
Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}
\]

Next we talked about Independent events. Two events are independent if \( P(A|B) = P(A) \)

Ex) Probability that card drawn in event A is a Jack given event B was the drawing of a red card.

\[
Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{2}{52} \div \frac{26}{52} = 1/13
\]

It was stated that if A and B are mutually exclusive -> (A \cup B) = 0 then A and B are never independent.

Various examples were then given to demonstrate independent events on a tree diagram.

The next item in the lecture was the gumdrop example. Every person in the class randomly picked a red or a green gumdrop from a bowl. It was found that out of the 13 females in the class, 5 chose red, 8 chose green. Out of the 48 males in the class, 24 chose red and 24 chose
green. The below tree diagram was made to compute the probabilities of each choice.