Problem 1: Valerie Variance is taking her "no-time limit" final exam. As always, the exam has an infinite number of questions and the instructor gives no partial credit. Valerie being a diligent student attempts ALL problems.

The number of problems that Valerie does CORRECTLY is described by a POISSON distribution with a mean of 8.0

She will PASS if she gets AT LEAST 8 problems correct.

GIVEN that she gets AT MOST 11 problems correct, what is the probability that she passes the exam. (Use table for Poisson distribution for calculations)

a) .8881  b) .5747  c) .4351  d) .6471  e) .4899

Problem 2: Students often have a problem understanding the meaning of an interaction between two factors. Consider the following two sets of means:

(i) Fac B

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
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<tbody>
<tr>
<td>31</td>
<td>35</td>
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<td>38</td>
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(ii) Fac A

<table>
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<td>48</td>
</tr>
<tr>
<td>28</td>
<td>55</td>
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Which, if any, of (i) and (ii) illustrate an interaction between factors A and B?
A. Neither (i) nor (ii).
B. (i)
C. (ii)
D. (i) and (ii)

Problem 3: For (i) in the previous problem, it appears that
A. Factors A and B are equally important
B. Neither factor A nor B are important
C. Factor A is more important than factor B
D. Factor B is more important than factor A
E. None of the above

Problem 4: If the form for a LARGE SAMPLE confidence interval for $\mu$ is given by

$$\bar{X} \pm 2.57(s / \sqrt{n})$$

what is the level of confidence?

A. 99.5%  B. 99%  C. 96%  D. 97%  E. 98%
**Problems 5 and 6 and 7:** The CDF \( F(x) = \Pr(X \leq x) \) of the vertical height that a North Pole reindeer can jump is given by (starts at \( X = 1 \) since legs are 1 meter long)

\[
F(x) = \begin{cases} 
0 & x < 1 \\
\frac{(x-1)^2}{4} & 1 \leq x < 3 \\
1 & x \geq 3
\end{cases}
\]

**Problem 5:**
Use this CDF to compute the 36\(^{th}\) percentile of \( X \) (to 3 decimal places)

a) 1.440  b) 2.200  c) 1.200  d) 2.600  e) 2.440

**Problem 6:** Find the probability that \( X \) is in the interval 0.5 to 1.5, that is \( \Pr(0.5 \leq X < 1.5) \) (note the lower limit carefully)

a) .5625  b) .0625  c) .0000  d) .1329  e) .3774

**Problem 7:** The mean of \( X \) is: hint differentiate to first get the density

a) 1.333  b) 2.333  c) 2.000  d) 2.173  e) 2.865

**Problem 8:** We are studying a population with unknown mean \( \mu \) and standard deviation \( \sigma \) that we guess is around 2. We plan to take a random sample and want to construct a 90\% confidence interval of length 3 (right endpoint MINUS left endpoint = 3). How large should the sample size be?

a. 34  b. 5  c. 16  d. 43  e. 27

**Problem 9:** The “Central Limit Theorem” is concerned with

a. A prediction interval for a future value of \( X \)
b. The estimation of \( \sigma \) by \( s \).
c. The approximate normal distribution of \( \bar{X} \)
d. The sampling distribution of the sample median.
e. The reliability of a series or parallel system.

**Problem 10:** What is the correct form of a 100(1 – \( \alpha \))\% SMALL-SAMPLE confidence interval for \( \mu \)

A. \( \bar{X} \pm (z_{\alpha/2})(s/\sqrt{n}) \)
B. \( \bar{X} \pm (t_{\alpha,n-1})(s/\sqrt{n}) \)
C. \( \bar{X} \pm (z_{\alpha,n-1})(s/\sqrt{n}) \)
D. \( \bar{X} \pm (t_{\alpha/2,n-1})(s/\sqrt{n}) \)
E. none of these
**Problems 11 to 14:** Over the past 27 years, the Lizard Lick Candy Company has produced an average of 57.0 pounds of jalapeño hedgehog birthday cake mix for Washington politicians each day. The records show that for the current year, based on 221 operating days, the following information was obtained:

- Sample mean \( \bar{X} = 55.6 \) pounds/day
- Sample standard deviation \( s = 2.1 \) pounds/day

It is desired to test whether the average daily production has **INCREASED SIGNIFICANTLY** over the current year.

**Problem 11:** Give the appropriate NULL (Ho) and ALTERNATIVE (Ha) hypotheses for this test

A. Ho : \( \mu \geq 55.6 \); Ha : \( \mu < 55.6 \)
B. Ho : \( \mu = 57.0 \); Ha : \( \mu \neq 57.0 \)
C. Ho : \( \mu = 55.6 \); Ha : \( \mu \neq 55.6 \)
D. Ho : \( \mu \leq 57.0 \); Ha : \( \mu > 57.0 \)
E. Ho : \( \mu \geq 57.0 \); Ha : \( \mu < 57.0 \)
F. Ho : \( \mu \leq 55.6 \); Ha : \( \mu > 55.6 \)

**Problem 12:** Determine the REJECTION region corresponding to \( \alpha = 0.05 \)

A. \( z < -1.645 \) or \( z > 1.645 \)
B. \( z < -1.96 \) or \( z > 1.96 \)
C. \( z < -1.645 \) or \( z > 1.645 \)
D. \( z > 1.645 \)
E. \( z < -1.645 \)

**Problem 13:** The value of the test statistic for this data is:

a) -7.333  b) -0.667  c) -14.362  d) -9.911  e) +0.667

**Problem 14:** A 99% confidence interval for the sample mean in the three previous problems is

a) (55.24, 55.96)  
 b) (55.27, 55.93)  
 c) (54.30, 56.90)  
 d) (56.64, 57.36)  
 e) (56.67, 57.33)
**Problem 15:** Establish the probability that the computed t from a sample of size 25 will fall below -0.685

a) .05  b) .75  c) .995  d) .25  e) none of these

**Problem 16:** The t distribution gets closer to the normal distribution as

a. the sample size increases  

b. the sample mean gets closer to zero  

c. the sample standard deviation decreases  

d. the median minus the mean becomes positive  

e. none of the above

**PROBLEMS 17, 18, 19:** The average kilowatt-hours per year of a particular home vacuum cleaner has been 46 kilowatt-hours per year. A random sample of 12 homes included in a planned study indicates these vacuum cleaners have a sample mean of 42 kilowatt hours/year with a sample standard deviation of 11.9 kilowatt hours/year. Is there evidence that there has been a decrease in the average yearly kilowatt-hour usage? Test the hypothesis $H_0: \mu \geq 46$ against $H_A: \mu < 46$.

**PROBLEM 17:** what is the observed t value associated with this test ?

A. +.336  B. -.336  C. -1.16  D. +1.16  E. -.697

**PROBLEM 18:** What is the critical value for this test at $\alpha = 0.05$ ?

A. -1.796  B. -2.201  C. +2.179  D. -1.782  E. +1.796

**PROBLEM 19:** What decision would one make at $\alpha = 0.05$ ?

A. the evidence is inconclusive  

B. a two-sided confidence interval for $\mu$ is needed in order to decide  

C. a $z$ test is needed since sample size is bigger than 10  

D. there is evidence AGAINST $H_0$; therefore REJECT $H_0$  

E. there is evidence FOR $H_0$; therefore ACCEPT $H_0$

**Problem 20:** space shuttle: For about $1$ billion in new space shuttle expenditures, NASA has proposed to install new heat pumps, power heads, heat exchangers, and combustion chambers. These will lower the mission catastrophe probability to 1 in 130. Assume that these changes are made. Calculate the probability of one or more catastrophes in the next: 140 missions

a) 1/130  b) .6608  c) 140/130  d) .2817  e) .3392
In a student project, how fast or the rate an image flashes or flickers before it is perceived as a continuous nonflickering image was measured as a function of the wavelength of the light (color) and the brightness, that is luminance level. 6 wavelengths and 6 luminance levels were used and all combinations of them. Each condition was run twice and the order of the 72 trials was completely randomized. The MFIT and mplot output is given below:

wav = wavelength, lum = luminance level, cff= rate of fusion the response

Mfit Data  Overall Mean of Y variable ff$cff = 64.847

Fitted main Effect of Y variable ff$cff by X variable ff$wav

<table>
<thead>
<tr>
<th>N</th>
<th>Main.Effect</th>
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<tbody>
<tr>
<td>415</td>
<td>12</td>
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<tr>
<td>472.5</td>
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<tr>
<td>532.5</td>
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<td>580</td>
<td>12</td>
</tr>
<tr>
<td>605</td>
<td>12</td>
</tr>
<tr>
<td>685</td>
<td>12</td>
</tr>
</tbody>
</table>

Fitted main Effect of Y variable ff$cff by X variable ff$lum

<table>
<thead>
<tr>
<th>N</th>
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<tbody>
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<td>22</td>
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<td>33</td>
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<td>42</td>
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<td>12</td>
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<tr>
<td>66</td>
<td>12</td>
</tr>
<tr>
<td>81</td>
<td>12</td>
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</tbody>
</table>

Table of 2-way Fitted Interaction Effects for ff$cff by X variables ff$wav and ff$lum

<table>
<thead>
<tr>
<th></th>
<th>22</th>
<th>33</th>
<th>42</th>
<th>57</th>
<th>66</th>
<th>81</th>
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</thead>
<tbody>
<tr>
<td>415</td>
<td>1.847</td>
<td>0.764</td>
<td>0.681</td>
<td>-2.403</td>
<td>-0.569</td>
<td>-0.319</td>
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<td>472.5</td>
<td>-0.153</td>
<td>0.264</td>
<td>-2.319</td>
<td>-0.403</td>
<td>0.931</td>
<td>1.681</td>
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<tr>
<td>532.5</td>
<td>-0.069</td>
<td>-0.653</td>
<td>0.764</td>
<td>0.681</td>
<td>0.514</td>
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<tr>
<td>580</td>
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<td>-0.319</td>
<td>0.097</td>
<td>1.014</td>
<td>0.347</td>
<td>-0.903</td>
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<tr>
<td>605</td>
<td>-1.986</td>
<td>0.931</td>
<td>1.347</td>
<td>0.764</td>
<td>-0.903</td>
<td>-0.153</td>
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<tr>
<td>685</td>
<td>0.597</td>
<td>-0.986</td>
<td>-0.569</td>
<td>0.347</td>
<td>-0.319</td>
<td>0.931</td>
</tr>
</tbody>
</table>
**Problem 21:** Using the Mfit data which statement is true:

a) wav is more important than lum, and lum is more important than interaction  
b) lum is more important than wav, and wav is more important than interaction  
c) interaction is more important than wav  
d) interaction is more important than lum  
e) the effects for each factor sum to zero, so more than 2 replications are needed

**Problem 22:** Using the Mfit data, the predicted CFF (flicker fusion rate) for wav = 580 and lum = 33 is:

a) 2.973  
d) 2.654  
c) 67.820  
d) 67.501  
e) 72.695

**PROBLEMS 23 and 24:** Five regression models were run on the data:

**MODEL 1** \( \text{lm} \gg \text{model } \text{cff} = \text{wav} + \text{lum} + \text{wav} \times \text{lum} + \text{wav}^2 + \text{lum}^2 \)

<table>
<thead>
<tr>
<th>Source</th>
<th>Parameter Estimate</th>
<th>Std. Error</th>
<th>t</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
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<td>8.193900000</td>
<td>-13.6798</td>
<td>0.0000000</td>
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<tr>
<td>wav</td>
<td>0.59536000</td>
<td>0.028382000</td>
<td>20.9767</td>
<td>0.0000000</td>
</tr>
<tr>
<td>lum</td>
<td>0.22356000</td>
<td>0.083382000</td>
<td>2.6811</td>
<td>0.0092585</td>
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<tr>
<td>wav*lum</td>
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<td>0.000109070</td>
<td>1.0712</td>
<td>0.2880000</td>
</tr>
<tr>
<td>wav^2</td>
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<td>-20.5050</td>
<td>0.0000000</td>
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<tr>
<td>lum^2</td>
<td>-0.00126430</td>
<td>0.000558020</td>
<td>-2.2657</td>
<td>0.0267580</td>
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</table>

**MODEL 2** model \( \log(\text{cff}) = \text{wav} + \text{lum} + \text{wav} \times \text{lum} + \text{wav}^2 + \text{lum}^2 \)

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<tbody>
<tr>
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<tr>
<td>wav</td>
<td>0.009413700</td>
<td>0.000426150</td>
<td>22.0901</td>
<td>0.000000000</td>
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<tr>
<td>lum</td>
<td>0.004135400</td>
<td>0.001252000</td>
<td>3.3031</td>
<td>0.001546900</td>
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<tr>
<td>wav*lum</td>
<td>8.3854e-007</td>
<td>1.6376e-006</td>
<td>5.1205</td>
<td>0.610330000</td>
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<tr>
<td>wav^2</td>
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<td>3.8045e-007</td>
<td>-21.4471</td>
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<tr>
<td>lum^2</td>
<td>-2.0918e-005</td>
<td>8.3786e-006</td>
<td>-2.49660</td>
<td>0.015044000</td>
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**MODEL 3** \( \text{lm} \gg \text{model } \text{cff} = \text{wav} + \text{lum} + \text{wav} \times \text{lum} \)

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<td>lum</td>
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<tr>
<td>wav*lum</td>
<td>0.00011683</td>
<td>0.000293250</td>
<td>0.39839</td>
<td>0.691590000</td>
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**MODEL 4** \[ \text{lm} >> \text{model } \sqrt{\text{cfl}} = \text{wav} + \text{lum} + \text{wav} \times \text{lum} + \text{wav}^2 + \text{lum}^2 \]

R-square 0.93553  
Standard Error 0.099806

Parameter Estimates  
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<td>0.000000000</td>
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<tr>
<td>lum</td>
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**MODEL 5** \[ \text{lm} >> \text{model } \log(\text{cfl}) = \text{wav} + \text{lum} + \text{wav} \times \text{lum} + \text{wav}^2 + \text{lum}^2 + \text{wav}^3 \]

R-square 0.94949  
Standard Error 0.022327

Parameter Estimates  
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**Problem 23:** this model gave the highest COEFFICIENT OF DETERMINATION or proportion of variation in response explained by the model

a) model 5  
b) model 4  
c) model 3  
d) model 2  
e) model 1

**Problem 24:** the interaction term wav*lum is:

a) significant in none of the models  
b) significant only in models 3 and 4  
c) significant only in models 2 and 5  
d) significant in all of the models  
e) an ANOVA is needed to determination significance of interaction terms  
f) an mplot is needed to determination significance of interaction terms
**Problem 25:** A baseball player is a 300 hitter if he gets a hit on 30% of his at bats or said another way with a probability of 0.30, assume that his successive at bats are independent. What is the probability that in his next 50 at bats that he will get between 15 and 19 hits INCLUSIVE (that is he gets 15 or 16 or 17 or 18 or 19 hits)?

a) 0.4468  b) 0.4684  c) 0.9152  d) 0.3460  e) 0.3830

**Problem 26:** A system has the following SIX independent components with reliabilities of FIVE of them written within the component boxes. The reliability of the system (to three decimal places) is 0.7100: FIND THE VALUE OF THE MYSTERY COMPONENT DENOTED BY ??

```
|-------| R1=0.9 |-------|
|       |        |       |
|-------| R2=0.8 |-------|
|       |        |       |
|-------| R3=0.7 |-------|
| R4=0.6|        | R5=0.5|
|       |        |       |
```

a) .9276  b) .9683  c) .7488  d) .8923  e) no such component exists

**Problem 27:** In a dice-tossing random experiment, a Red die and a Green die are thrown independently. Consider as a random variable, X, the range in showing dots (that is the largest value-smallest value). For example if the red die is 6 and the green die is 4, then the random variable has a value of 2 in that experiment. Calculate the probability, \( \Pr(X=3) = \)

a) .111  b) .167  c) .0556  d) .278  e) .222

**Problem 28:** Suppose that we want a 95% confidence interval for \( \pi \) = proportion of defectives if we observe 12 defectives in 60 items tested (to 3 decimals).

a. (.004, .396)
b. (.119, .281)
c. (.099, .301)
d. (.068, .332)
e. (.047, .353)
Problems 29 and 30: Uncle Basil
In the ever popular TV soap opera, *As My Stomach Turns*, Boston Bart is planning to murder his rich Uncle Basil in hopes of claiming his inheritance a bit early. Hoping to take advantage of his Uncle’s love for lavish desserts, Bart puts:
- Rat poison in the cherries flambé
- Cyanide in the chocolate mouse

The probability of rat poison being fatal is 0.60, while cyanide provides less suspense to the plot since cyanide is fatal 90% of the time. On this TV show, rich uncles ONLY die due to some sort of poison.

Based on other dinners with his Uncle, Bart knows that his Uncle orders:
- cherries flambé 30% of the time
- chocolate mouse 60% of the time, and
- 10% of the time orders something else or skips dessert.

Problem 29: What is the probability that the Uncle dies on the next episode:

a) 0.90  b) 0.63  c) 0.72  d) 0.54  e) 0.84

Problem 30: You missed the soap opera for a week due to exams, in the next episode you find out that Uncle Basil is dead. WHAT IS THE PROBABILITY THAT IS WAS THE CHOCOLATE MOUSE THAT KILLED HIM?

a) 0.25  b) 0.75  c) 0.54  d) 0.60  e) 0.84

Problem 31: The elongation of a steel bar under a particular tensile load may be assumed to be normally distributed, with a mean of .06 in. and standard deviation of .008 in. A sample of n=100 bars is subjected to the test. Find the probability that the sample mean elongation is between .059 in. and .061 in.

a) 0.8944  b) 0.7888  c) 0.0956  d) 0.1034  e) none of these

Problem 32: At a local circus, “Terrific Tomoko” tames tumultuous tigers. Assume the time it takes her to tame a tiger has an exponential probability density with a mean of 2 days. What is the probability that the next tiger tamed with take AT LEAST 2 days?

a) 0.3679  b) 0.6321  c) 0.0183  d) 0.9817  e) none of these

Problem 33: The volume of liquid in a can of Dr, Extreme from Harris Teeter in Cameron Village, X, is approximately a NORMAL random variable with \( \mu_X = 355 \) ml. and \( \sigma_X^2 = 25 \text{ ml}^2 \). Find the \( 99^{th} \) percentile for the volume in the can

a) 413.25  b) 367.85  c) 419.25  d) 366.65  e) 373.62
5 point EXTRA CREDIT:

List something that would have help improve the course for next semester, it can be about webassign, project, quizzes, office hours, etc.

Formulas that could be of use:

\[ \int x^n \, dx = \frac{x^{n+1}}{n+1} \]

The Weibull probability density

\[ f(t) = \lambda \beta (\lambda t)^{\beta-1} \exp(-\lambda t^\beta) \quad \text{for } t \geq 0 \]

The binomial(n; \pi) probability mass function is

\[ Pr(X = k) = \frac{n!}{k!(n-k)!} \pi^k (1-\pi)^{n-k} \]

\[ k = 0, 1, \ldots, n \]

The Poisson probability mass function for mean process rate and time span t is

\[ Pr(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \]

\[ k = 0, 1, 2, \ldots \]