Statistics of Credit Risk Management in Financial Derivatives

Peter Bloomfield
Department of Statistics
North Carolina State University

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Overview

- derivatives
- credit risk
- risk mitigation
- use of collateral
- intermediation
- capital adequacy
- portfolio management
Derivatives:

“A financial instrument whose value derives from some underlying asset”

Futures, options, etc.

*Over-the-counter* derivatives contract:

- two-party agreement to exchange cash flows;
- lasts for a specified period of time;
- cash flows determined by future market rates, e.g. interest rates.
Example: Interest rate swap

- ABC Corp agrees to pay to XYZ Bank a fixed interest rate of 7% on a principal amount of $10,000,000 for a term of 5 years.
- XYZ Bank agrees to pay to ABC Corp a “floating” interest rate\(^1\) on the same principal amount for the same term.

1. E.g. LIBOR (London InterBank Offered Rate), currently 5.97% for 6-month deposits, + a spread.
Example: Interest rate swap (contd.)

- “Principal amount” does not change hands.
- Only a net payment is made on any payment date.
Marking to Market

Mark-to-Market value:
- net present value (NPV) of remaining cash flows.

Mark-to-market value changes over time:
- with market rates;
- as deal matures.

E.g. If rates rise, ABC expects to receive net payments from XYZ. ABC is “in the money”.
Credit Risk

Default Risk:

• one party may fail to honor contract;
• standard contract requires prompt settlement of NPV (of this and all other contracts);
• other party is an unsecured creditor, unless security is provided.
How much risk?

Party A loses money if:

• Party B defaults, and
• A is *in the money*, and
• recovery is less than 100%
### One-year transition probabilities (%):

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<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>D</th>
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Cumulative default rates (%, Markov model):

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Historical multi-year rates (%):  
(Generally higher)

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<td>44.20</td>
<td>45.10</td>
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Mitigating Risk: Collateral

Party A is in the money.

Party A requests collateral (cash or Treasury securities) from Party B.

How much?

- Market value of collateral = $C$;
- B defaults at most $d$ days later;
- Want $P(C + \delta C > NPV + \delta NPV) > 1 - \varepsilon$. 
Mitigating Risk: Collateral (contd.)

Must:
- specify \( d \) and \( \varepsilon \);
- find distribution of \( \delta NPV - \delta C = \delta(NPV - C) \)

Typically:
- assume normal distribution
- \( \mu = 0, \sigma = \text{calculated SD of } \delta(NPV - C) \)
- set \( C = NPV + k\sigma, \ k = 2 \text{ or } 3. \)

Often SD of \( \delta(NPV - C) \) is replaced by

\[
\text{SD of } \delta(NPV) + \text{SD of } \delta(C)
\]
Mitigating Risk: Intermediation

Possible Intermediaries:

- third party;
- Credit-enhanced Derivative Product Company ("DPC") wholly owned by Party B.
Credit Enhancement

How does DPC rate higher than B?

- bankruptcy-remote;
- no market risk;
- no credit exposure to B (collateral);
- enough capital to withstand other credit losses.
DPC Structures

Continuation Vehicle:
- survives insolvency of parent;
- Contingent Manager steps in;
- contracts run to maturity.

Termination Vehicle:
- all contracts terminate on insolvency of parent;
- NPV exchanged promptly with all counterparties.
Capital Adequacy

Rating agencies require

\[ P(\text{credit losses} > \text{capital}) > 1 - \varepsilon \]

or similar.

Overall credit loss = \( \sum \) Counterparty loss

Counterparty loss = \( \max\{0, \text{NPV}(\tau)\} \)

\( \tau = \text{time of counterparty event} \)
Capital Adequacy

Assumptions:
1. $\text{NPV}(t)$ -
   - depends on market rates at $t$;
   - not independent across counterparties.
2. $\tau$ –
   - need joint distribution;
   - $\tau = \min\{\tau(\text{counterparty}), \tau(\text{sovereign})\}$
   - independence plausible (perhaps conditionally on market conditions).
Capital Adequacy

Computational strategy:

- condition on market path;
- conditional distribution of credit loss depends on distribution of τ’s (and path);
- conditional independence => distribution of sum is convolution;
- get unconditional distribution by averaging over market paths (simulation);
Optimization

Capital and collateral cost money.

Required levels of both depend on portfolio of contracts:
- credit ratings of counterparties;
- sensitivity of NPV’s to market fluctuations.

Portfolio can be managed to control costs.
Summary

Modern quantitative methods play a role in:

- mitigating credit risk, either through use of collateral or through structuring a special purpose intermediary;
- efficiently managing a portfolio of derivatives.