Statistical Issues in Managing the Credit Risk of Derivatives

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Workshop on Mathematical Finance

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Overview

• derivatives
• credit risk
• risk mitigation
• use of collateral
• intermediation
• capital adequacy
• portfolio management
Derivatives:

Over-the-counter derivatives contract:

- two-party agreement to exchange cash flows;
- lasts for a specified period of time;
- cash flows determined by future market rates, e.g. interest rates.
Marking to Market

Mark-to-Market value:
• net present value (NPV) of remaining cash flows.

Mark-to-market value changes over time:
• with market rates;
• as deal matures.
Credit Risk

Default Risk:

- one party may fail to honor contract;
- standard contract requires prompt settlement of NPV (of this and all other contracts);
- other party is an unsecured creditor, unless security is provided.
How much risk?

Party A loses money if:

- Party B defaults, and
- NPV to A is positive, and
- recovery is less than 100%
### One-year transition probabilities (%):

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<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
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<th>D</th>
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Cumulative default rates (%, Markov model):

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Historical multi-year rates are higher.
Mitigating Risk: Collateral

Party A’s NPV > 0.

Party A requests collateral (Treasury securities) from Party B.

How much?

- Market value of collateral = C;
- B defaults at most \( d \) days later;
- Want \( P(C + \delta C > NPV + \delta NPV) > 1 - \varepsilon \).

Must:

- specify \( d \) and \( \varepsilon \);
- find distribution of \( \delta NPV - \delta C = \delta(NPV - C) \).
Mitigating Risk: Intermediation

Possible Intermediaries:
- third party;
- Credit-enhanced Derivative Product Company ("DPC") wholly owned by Party B.
Credit Enhancement

How does DPC rate higher than B?

- bankruptcy-remote;
- no market risk;
- no credit exposure to B (collateral);
- enough capital to withstand other credit losses.
DPC Structures

Continuation Vehicle:
- survives insolvency of parent;
- Contingent Manager steps in;
- contracts run to maturity.

Termination Vehicle:
- all contracts terminate on insolvency of parent;
- NPV exchanged promptly with all counterparties.
Capital Adequacy

Rating agencies require

\[ P(\text{credit losses} > \text{capital}) > 1 - \varepsilon \]

or similar.

Overall credit loss = \( \sum \) Counterparty loss

Counterparty loss = \( \min\{0, \text{NPV}(\tau)\} \)

\( \tau = \text{time of counterparty event} \)
Capital Adequacy

Assumptions:
1. $\text{NPV}(t)$ -
   - depends on market rates at $t$;
   - not independent across counterparties.
2. $\tau$ -
   - need joint distribution;
   - $\tau = \min\{\tau(\text{counterparty}), \tau(\text{sovereign})\}$
   - independence plausible (perhaps conditionally on market conditions).
Capital Adequacy

Computational strategy:
- condition on market path;
- conditional distribution of credit loss depends on distribution of τ’s (and path);
- conditional independence => distribution of sum is convolution;
- get unconditional distribution by averaging over market paths (simulation);
Optimization

Capital and collateral cost money.

Required levels of both depend on portfolio of contracts:
  • credit ratings of counterparties;
  • sensitivity of NPV’s to market fluctuations.

Portfolio can be managed to control costs.
Summary

Probability modelling and statistical analysis play role in:

- mitigating credit risk, either through use of collateral or through structuring a special purpose intermediary;
- efficiently managing a portfolio of derivatives.