Sample Questions for Final Exam
ST422 - Introduction to Mathematical Statistics-II

The ACTUAL exam will consists of less number of problems.

1. If Y_1 and Y_2 have a joint distribution given by

$$\begin{aligned} f(y_1, y_2) &= \frac{1}{2} y_1 y_2 \text{ if } 0 \leq y_2 \leq y_1 \leq 2 \\ &= 0 \text{ elsewhere} \end{aligned}$$

   (a) Find the marginal distributions of Y_1 and Y_2.

   (b) Are Y_1 and Y_2 independent? Find $Cov(Y_1, Y_2)$.

   (c) Find the conditional distribution of Y_2 given $Y_1 = y_1$.

   (d) Find the values of $E(Y_2|Y_1 = 1)$ and $Var(Y_2|Y_1 = 1)$.

   (e) What is the density of $U = Y_1 - Y_2$.

2. If $F(\cdot)$ is a probability distribution function, i.e. $F(y) = \Pr[Y \leq y]$.:

   (a) Is $F(y_1, y_2) = F(y_1) + F(y_2)$ a joint probability distribution function?

   (b) Is $F(y_1, y_2) = F(y_1)F(y_2)$ a joint probability distribution function?

   (c) Is $F(y_1, y_2) = \max\{F(y_1), F(y_2)\}$ a joint probability distribution function?

   (d) Is $F(y_1, y_2) = \min\{F(y_1), F(y_2)\}$ a joint probability distribution function?

3. Three fair coins are tossed. Let
   Y_1 = number of heads on the first two coins, and
   Y_2 = number of tails on the last two coins.

   (a) Find the joint distribution of Y_1 and Y_2.

   (b) Find the conditional distribution of Y_2 given that $Y_1 = 1$.

   (c) Find $Cov[Y_1, Y_2]$.

   (d) Find the distribution of $U = Y_2 - Y_1$.

4. Prove or disprove: If $E[Y_2|Y_1] = Y_1, E[Y_1|Y_2] = Y_2$ and both $E[Y_1^2]$ and $E[Y_2^2]$ are finite, then $\Pr[Y_1 = Y_2] = 1$.

5. Projectiles are fired at the origin of an xy-plane. Assume that the point which is hit, say $X, Y)$, consists of a pair of independent standard normal random variables. For two projectiles fired independently of one another, let $X_1, Y_1)$ and (X_2, Y_2) represent the points which are hit, and let Z be the distance between them. Find the distribution of the Z^2.

6. Let $Y_1, Y_2 \sim^{iid} N(0, 1)$. Find the mgf of $Y = Y_1 Y_2$.

7. If $Y_1, Y_2, \ldots, Y_n$ are independent Poisson random variables, show that the conditional distribution of Y_1, given $\sum_{i=1}^n Y_i$ is Binomial.

8. If $Y_1, Y_2, \ldots, Y_n \sim^{iid} Exp(\theta)$, what is the distribution of $U = \sum_{i=1}^n Y_i$?

9. If $Y_1, Y_2, \ldots, Y_n \sim^{iid} Geometric(p)$, what is the distribution of $U = \sum_{i=1}^n Y_i$?

10. If $Y_i \sim^{indep} \chi^2_{\nu_i}, i = 1, 2, \ldots, n$, what is the distribution of $U = \sum_{i=1}^n Y_i$?

11. If $Y_i \sim^{indep} N(\mu_i, \sigma_i^2), i = 1, 2, \ldots, n$, what is the distribution of $U = \sum_{i=1}^n Y_i$?

12. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from the density:

$$\begin{aligned} f(y) &= \theta y^{-2} \text{ if } 0 < \theta \leq y \\ &= 0 \text{ elsewhere} \end{aligned}$$

(a) Find a sufficient statistic for θ.

(b) Find the MLE of θ.

(c) Does the method of moment estimate of θ exist? If yes, find it. If no, why?

13. Let $Y_1, Y_2, \ldots, Y_n \sim^{iid} Poisson(\lambda)$. Define $\theta = (1 + \lambda)e^{-\lambda}$.

(a) Find a sufficient statistic for θ.

(b) Find the MLE of θ.

(c) Find an unbiased estimator of θ.

(d) Find the UMVUE of θ.

14. What is the probability that the length of a t-confidence interval for μ when sampling from a normal distribution will be less than σ for samples of size 20?

15. Suppose that 175 heads resulted from 400 tosses of a coin. Find a 90% confidence interval for the probability of a head. Find a 99% confidence interval. Does this appear to be a true coin?

16. Additional suggested problems from the text: **5.100, 5.103, 6.30, 6.63, 6.67, 6.71, 7.49, 8.103, 8.108, 8.114, 9.28, 9.44, 9.59, 9.69, 9.74, 10.14, 10.18, 10.27 & 10.46**

    (*I strongly recommend you to solve all these additional problems*)

Solutions to above problems will NOT be posted on the web. However, I'll solve the problems in class.