

Sample questions for Final Exam ST370 - Probability and Statistics for Engineers

Final Exam will have a total of 10 questions similar to the following problems.

1. A sample consisting of 4 pieces of luggage was selected from among those checked at an airline counter, yielding the following data on $X = \text{weight (lb)}$:

$$X_1 = 33.5, X_2 = 27.3, X_3 = 36.7, X_4 = 30.5$$

Suppose one more piece is selected; denote its weight by X_5 . Find a value of X_5 so that $\bar{X} = \text{median}$.

2. Give an example for each of the following data types:
(a) nominal (b) ordinal (c) interval (d) ratio.
3. The first four deviations from the mean in a sample of 5 reaction times were 0.3, 0.9, 1.0, and 1.3. What is the fifth deviation from the mean ?
4. Is the following statements correct?

- (a) "A correlation coefficient (r) of zero implies that no relationship exists between the two variables under study." Explain why or why not.
- (b) "The slope b of the least squares line always has the same sign as does the correlation r ." Explain why or why not.

5. From a sample data on the reaction time (Y in hours) and retort chamber temperature (X in $^{\circ}\text{F}$), the following summary values, based on 15 observations were obtained:

$$\sum X = 3525, \sum Y = 43.1, \sum XY = 10058, \sum X^2 = 835375, \sum Y^2 = 124.63.$$

Compute the,

- (a) correlation coefficient r ,
- (b) slope and intercept of the estimated line $\hat{Y} = a + bX$; and
- (c) coefficient of determination R^2 (Multiple- R^2).

Based on your estimated line in part (b), what reaction time would you predict for a temperature of 240°F ?

6. A card is selected from a well-mixed deck. Let E be the event that the card is a heart, and let F be the event that the card is a face card.

(a) Determine $\Pr[E]$ and $\Pr[F]$.

(b) Determine $\Pr[E | F]$ and $\Pr[F | E]$.

(c) Are E and F independent? Are E and F mutually disjoint?

7. Let the following be a probability distribution for X , the number of courses for which a randomly selected student is registered:

x	1	2	3	4	5	6	7
p(x)	.02	.03	.09	.25	.40	.16	.05

(a) Calculate the expected value of X i.e. $E(X)$.

(b) Calculate the standard deviation of X i.e. $SD(X)$.

(c) What possible values of X are within two standard deviations of the expected value? What is the probability that the observed value of X is within two standard deviations of the expected value?

8. A manufacturer of camera flash bars notes that defective flash bulbs are sometimes produced. If the probability that any given bulb will be defective is .05, what is the probability that there are no defective flashes on a flash bar containing ten bulbs? What is the probability that at most one of the ten bulbs is defective?

9. The air pressure in a randomly selected tire put on a certain model new car is normally distributed with mean value 31 lb/in.² and standard deviation .2 lb/in.²

(a) What is the probability that the pressure for a randomly selected tire exceeds 30.5 lb/in.²?

(b) What is the probability that the pressure for a randomly selected tire is between 30.5 and 31.5 lb/in.²? Is between 30 and 32 lb/in.²?

(c) Suppose a tire is classed as underflated if its pressure is less than 30.4 lb/in.². What is the probability that at least one of the four tires on a car is underflated?

10. Suppose that a sample of size 100 is to be drawn from a population with standard deviation 10. What is the probability that the sample mean will be within 2 of the value of μ = true population mean?
11. A hot-tub manufacturer advertises that with its heating equipment, a temperature of 100°F can be achieved in at most 15 min. A random sample of n tubs is to be selected and the time necessary to achieve a 100°F is to be determined for each tub. It is known that time required to achieve 100°F lie in the range of 10 to 30 minutes.
- (a) Find the sample size n in order to achieve a precision of ± 1 with 95% reliability.
- (b) Suppose a survey based on 40 hot-tubs yielded a sample mean of 17.5 minutes with a standard deviation of 3.9 minutes. Compute the 95% C.I. for the true mean time required to achieve 100°F. Does this data cast doubt on the company's claim ?
12. If $p(x)$ is the probability mass function of a discrete random variable X , what are the two properties **must** p possess ? (Note : $p(x) = \Pr[X = x]$.)
13. Decide if the following statements are True or False.
- (a) A 95% confidence interval for μ will contain approximately 95% of the observed data.
- (b) If two random variables, X and Y are independent then $Corr(X, Y) = 0$.
- (c) If $Corr(X, Y) = 0$, then X and Y are independent.
- (d) $E(a_1X_1 + a_2X_2) = a_1E(X_1) + a_2E(X_2)$.
- (e) $Var(a_1X_1 + a_2X_2) = a_1Var(X_1) + a_2Var(X_2)$.
- (f) Let X be a random variable with mean μ and variance σ^2 .
- i) $\mu = \sigma^2$, if X has a Poisson distribution.
- ii) $\mu > \sigma^2$, if X has a Binomial distribution.
14. The continuous random variable X has density function

$$f(x) = \begin{cases} \frac{1}{18}(x^2 - 1) & \text{if } 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(1 < X < 3)$. Find $E(X)$ and $SD(X)$.

15. Discuss how each of the following factors affects the width of the large-sample confidence interval for μ when σ is known.
- (a) Confidence level.
 - (b) Sample size.
 - (c) Population standard deviation.
16. A manufacturer of video recorder tapes sells tapes labeled as giving 6h of playing time. If the mean and standard deviation of 64 observed playing times are 352min and 8 min, respectively,
- (a) Construct a 99% C.I. for μ , the true average playing time for 6-h tapes. Does this data provide sufficient evidence to conclude that the true mean playing time is 6 h?
 - (b) Suppose you want to conduct another survey. How many tapes should be tested if you wish to estimate μ to within 5 min with 95% confidence?
17. A consumer group is interested in estimating the proportion of packages of ground beef sold at a particular store that have an actual fat content exceeding the fat content content stated on the label. How many packages of ground beef should be tested in order to estimate this proportion to within .05 with 95% confidence?
18. Suggested problems from the chapter 10:
10-27, 10-37, 10-39, 10-41, 10-60, 10-61 and 10-63.

Solutions to above problems will NOT be posted on the web. However, I'll solve the problems in class. Bring the list of formula and tables A, C, D, E & G

It would be a good idea to review the problems of previous tests and their sample questions.