

# LECTURE 9 Draft

## CLOSED CAPTURE-RECAPTURE

### MODELS

More on Closed Models Today

#### --MARK

- Lincoln-Petersen example (one and two groups)

- Taxi Cab Example

#### --CAPTURE

- From MARK

- From Web

Table 3.1. Capture-recapture models for closed populations that allow for unequal capture probabilities.

Monograph with minor changes.

<i>Model*</i>	<i>Source of variation</i>	<i>in</i>	<i>capture probability</i>
	Heterogeneity	Trap response	Time
$M_o$			Estimator availability yes
$M_h$	X <sup>a</sup>		yes
$M_b$		X	yes
$M_{bh}$	X	X	yes
$M_t$			X yes
$M_{th}$	X		X yes
$M_{tb}$		X	X yes
$M_{tbh}$	X	X	X no

\*This set of 8 models comes from Otis et al. (1978).

<sup>a</sup>Xs denote the sources of variation in capture probability incorporated in the models.

# CAPTURE

- Classic closed population models of Otis et al. (1978), but with some updating for new estimators.
- Contains a model selection procedure.
- Can be run on the web from Patuxent software archive.
- Copy of program can be downloaded from Patuxent software archive.
- Can also run from MARK.
- I will show you both ways to access CAPTURE later in Lecture

# MARK

- User friendly windows based program for capture-recapture, telemetry and band return models.
- Many options
- Can run CAPTURE and POPAN from MARK
- Uses AIC for model selection
- Allows multiple groups, age classes, multi-state extension, covariates
- Can download from their web site. Can also download an online book and other resources.

<http://welcome.warnercnr.colostate.edu/~gwhite/mark/mark.htm>

<http://www.phidot.org/software/mark/docs/book/>

# Use of MARK

- Rabbit data (2 periods LP), Insect data (2 periods and 2 groups)
- Taxi cab data (10 periods)
- Input format and syntax
- Use the parameter index matrices (PIMS) to create specific models to run.(M0, Mt, Mb)
- I will show you how the AIC is used to compare models and select the best one. Then I will show you how to look at the output files for a chosen model
- I will show how to switch to looking at CAPTURE output when you are in a MARK analysis if you want to use both at once.

# Simple LP model in MARK

## RABBIT EXAMPLE

Data format

Setting up the parameter constraints  
using PIMS

Model Selection

Chosen Model Output

# MARK: RABBIT EXAMPLE

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Data format –Notepad File with --.INP extension

Capture history , No animals.

Note; at end of line.

11 7;

10 77;

01 7;

Note: Comments /\*Title\*/

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# MARK: RABBIT EXAMPLE

Setting up the parameter constraints using PIMS.

$p_1, p_2$  first capture prob time 1 & 2

$p_3$ , recapture prob time 2 (c2)

$p_4$ , population size  $N$

$M(t)$   $p_1, p_2=p_3, N-3$  parameter model

$M(0)$   $p_1=p_2=p_3, N-2$  parameter model

$M(b)$   $p_1=p_2, p_3, N-3$  parameter model

# MARK: RABBIT EXAMPLE

Model Selection- We fitted  $M(t)$  and  $M(0)$ .

Rabbit Analyses for 506

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Model	AICc	AICc	Delta Weight	AICc Likelihood	Model #Par	Deviance
{M(t)}	-536.099	0.00	1.00000	1.0000	3.0000	5.548
{M(0)}	-469.416	66.68	0.00000	0.0000	2.0000	74.299

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Note- How  $M(t)$  so much favoured. Lets look at each output.

# MARK: RABBIT EXAMPLE

## Model Output (Mt)

Rabbit Analyses for 506

Real Function Parameters of {M(t)}

95% Confidence Interval

Parameter	Estimate	Standard Error	Lower	Upper
1:p	0.5200943	0.1359513	0.2714729	0.7591460
2:p	0.0866824	0.0309946	0.0422018	0.1697371
3:N	161.50918	40.414581	115.80591	291.41774

Note estimate of N we get from the Lincoln –Peterson.

# MARK: RABBIT EXAMPLE

## Model Output $M(0)$

Rabbit Analyses for 506

Real Function Parameters of  $\{M(0)\}$

95% Confidence Interval

Parameter	Estimate	Standard Error	Lower	Upper
1:p	0.1456141	0.0489486	0.0730623	0.2692812
2:N	336.50590	108.66627	198.15572	653.48186

Note-Crazy Estimate of N

# MARK: RABBIT EXAMPLE

## Notes

$M(0)$  doesn't really make sense in this example.

$M(b)$  doesn't really make sense in this example.

Why: Two different sampling methods used.

# Two Groups LP model in MARK

## INSECT EXAMPLE

Data format

Setting up the parameter constraints  
using PIMS

Model Selection

Output for Selected Model

# MARK

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## INSECT EXAMPLE

Data format –Notepad File

Capture history , No animal gp1(M), gp2(F).

Note; at end of line.

10 63 76;

01 25 37;

11 14 19;

---

# MARK

## INSECT EXAMPLE

Setting up the parameter constraints using PIMS.

p1,p2 gp 1 first capture prob time 1& 2

p3,p4 gp 2 first capture prob time 1& 2

p5, group 1 recapture prob (p5=p2 here)

p6, group 2 recapture prob (p6=p4 here)

# MARK

## INSECT EXAMPLE

Setting up the parameter constraints using PIMS

For LP sep sexes

p1,p2 gp 1 capture prob time 1& 2

p3,p4 gp 2 capture prob time 1& 2

Combined Sexes Restricted Model

$p1=p3$  and  $p2=p4$

## P picivorus Male and Female Data

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		Delta	AICc	Model			
Model	AICc	AICc	Weight	Likelihood	#Par	Deviance	
{LP Comb}	-1318.987	0.00	0.86021	1.0000	4.0000	12.354	
{LP Both}	-1315.353	3.63	0.13979	0.1625	6.0000	11.891	

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## P picivorus Male and Female Data

{LP Both Sexes} {LP Comb Sexes}

Parameter	Estimate	SE	Estimate (SE)	
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1:p (M1)	0.37	0.077	0.35	0.049
2:p (M2)	0.18	0.044	0.19	0.030
3:p (F1)	0.34	0.063	0.35	0.049
4:p (F2)	0.20	0.041	0.19	0.030
5:N (M)	210.86	40.54	212.68	29.09
6:N (F)	276.53	45.90	275.38	36.45

Note- 1&3 and 2&4-Lets simplify by putting them equal in the combined analyses. Gains in precision!

# The Estimates from Best Model

P Picivorus Male and Female Data  
LP Sexes Combined

Parameter	Estimate	St Error
1:p (M)	0.3524099	0.0492986
2:p (F)	0.1946450	0.0303306
3:N (M)	212.68334	29.093079
4:N (F)	275.38463	36.456268

# Taxi Cab Example: Background

- $N=420$  is the true popn size and there were 10 sampling occasions.
- The data set is in the MARK examples as File Carothers Scheme A.inp.
- I suggest you save as another .inp file if you want to try various analyses as otherwise you will wipe out the analyses Gary White has already done.

# Taxi cab Data: Part of Input File

```
/* Carothers (1973) Scheme A taxicab data, true population =  
420 occasions=10 groups=1 */  
/* 1 */ 0001000001 1;  
/* 2 */ 1000100000 1;  
/* 3 */ 0000010000 1;  
/* 4 */ 0001001000 1;  
/* 5 */ 0000000001 1;  
/* 6 */ 1101010000 1;  
/* 7 */ 0010000000 1;  
/* 8 */ 0101000010 1;
```

There are 283 records-one for each taxi cabs history, note the format used for comments, also this one does not use summary format like the rabbit and insect data.

# Taxi Cab Example- PIMS M(t)

$M_t$  PIMS Structure

p 1,2,3,4,5,6,7,8,9,10

c 2,3,4,5,6,7,8,9,10

N 11

Note- In the full model c PIMS 11,12,13,14,  
15, 16,17,18,19 with N PIM 20

# Taxi Cab Example- PIMS M(0)

$M_0$  PIMS Structure

p 1,1,1,1,1,1,1,1,1,1

c 1,1,1,1,1,1,1,1,1

N 2

Note- Full Model 1-10 all 1, 11-19 all, 20 is 2.

# Taxi Cab Example: $M(b)$

$M_b$  (Not really needed but shows interesting features)

PIMS Structure

$p$  1111111111

$c$  2222222222

$N$  3

# CAPTURE

- Classic closed population models of Otis et al. (1978), but with some updating for new estimators.
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# Use of CAPTURE from MARK

- I will demonstrate this in class using the taxi cab data as an example
- You have to access the output in an unusual way by going to the tests tab in an output window.
- It will be more natural to look at MARK first.

# Summary of Some Output

## From CAPTURE (Inside MARK)

- $M(0)$  368 with standard error 14.4896 (underestimates when there is heterogeneity).
- Model  $M(h)$  Suggested for use here from old Model selection Procedure
- Jackknife 471 with standard error 36.32
- Chao 407 with standard error 27.42

## From MARK Directly with PIMS

- Model  $M(0)$  368 with standard error 14.4896 (underestimates when there is heterogeneity).
- Can also fit  $M(0)$  and  $M(t)$  with PIMS.
- Can also fit Finite Mixture approach ( for heterogeneity but did not work too well here. Nhat 463 with huge SE of 273.88!) using the “Closed Captures” and then “with Heterogeneity” procedure. Will not insist you know this one in detail.

# USE of CAPTURE on the Web

- <http://www.mbr.pwrc.usgs.gov/software/capture.html>
- See hardcopy handout from last class on the input formats and output for three examples.
  - Rabbit data-Darroch (Mt)
  - Microtus data-Jackknife (Mh)
  - Removal data-Zippen (Related to Mb)

# USE of CAPTURE on the Web

Would only recommend you use this approach for very simple examples

- Rabbit data-Darroch (Mt, Lincoln-Petersen)

Not very suitable for the Microtus data because we cant fit all the models we would like

- Microtus data-I showed you the Jackknife (Mh)

# USE of CAPTURE on the Web

You could use this approach for removal data which I will now discuss if I have time.  
(Zippen, Removal procedures)

## More on Model $M_b$

- Trap Response is a difficult issue.
- Trap Response models are similar statistically to removal models although obviously biologically they are different.

# The Trap Response Model and Removal

- Under Model  $M_b$ , animals do not contribute any information for population size estimation after first capture. They can be thought of as having been “removed” from the unmarked population. Thus this model is statistically equivalent to the removal model where animals are actually physically removed.
- What I am saying is that one can use CAPTURE and models  $M_b$ ,  $M_{bh}$  for two situations
  - (a) Capture-recapture with trap response (without/with heterogeneity).
  - (b) True removal data (without/with heterogeneity).

# REMOVAL DATA

```
graph TD; A[REMOVAL DATA] --> B[Non-selective Removals (14.4)]; A --> C[Selective Removals Change-in-ratio(14.5)];
```

**Non-selective**

**Removals (14.4)**

**or**

**Selective Removals**

**Change-in-ratio(14.5)**

# Non-selective Removals

```
graph TD; A[Non-selective Removals] --> B[Equal Sampling Effort]; A --> C[Unequal Sampling Effort]; B --> D[Controlled studies]; C --> E[Uncontrolled Studies]; D --> F[REMOVAL MODELS]; E --> G[CATCH EFFORT MODELS]; F --> H[Closed]; G --> I[Closed];
```

## Equal Sampling Effort

### Controlled studies

- Small mammal trap grids
- Electrofishing in small streams

### REMOVAL MODELS

Closed

## Unequal Sampling Effort

### Uncontrolled Studies

- Commercial fisheries
- Hunter species with control through complete check stations

### CATCH EFFORT MODELS

Closed

## Removal Methods (non-selective)

- ❖ These are methods of estimating abundance when animals are removed from the population (but not marked nor replaced).
- ❖ Usually we consider them to be short term and the animals are not replaced.

### Examples

- small mammal grids
- electrofishing in a stream

# Closed Population with Constant Effort

## 1. Two Sample Case:

$n_1$  – removed at time 1

$n_2$  – removed at time 2

### Estimation

$$n_1 / N \approx n_2 / (N - n_1)$$

$$\hat{N} = \frac{n_1^2}{(n_1 - n_2)}$$

$$\hat{p} = \frac{(n_1 - n_2)}{n_1}$$

# Closed Population with Constant Effort

## 1. Two Sample Case:

### Assumptions:

- Closure
- Capture probability ( $p$ ) is constant over animals
- Capture probability constant over time

Failure: Sometimes if  $p$  is low even if the assumptions are valid, the method will fail if  $n_1 = n_2$  or  $n_1 < n_2$

# Closed Population with Constant Effort

## 1. Two Sample Case:

Example- *Maecolaspis flavida* –a beetle - removal data from first two sets of sweeps of a net. Each set is four sweeps. Menhinick(1963) Ecology 44, 617-621.

$$\hat{N} = \frac{n_1^2}{(n_1 - n_2)} = \frac{135^2}{135 - 76} = \frac{135^2}{59} = 308.90$$

$$\hat{p} = \frac{(n_1 - n_2)}{n_1} = \frac{135 - 76}{135} = \frac{59}{135} = 0.4370$$

# Closed Population with Constant Effort

Example- *Maecolaspis flavida* –a beetle - removal data from first two sets of sweeps of a net. Each set is four sweeps. Menhinick(1963) Ecology 44, 617-621

$$\begin{aligned} \text{SE}(\hat{N}) &= \sqrt{\frac{n_1^2 n_2^2 (n_1 + n_2)}{(n_1 - n_2)^4}} \\ &= \sqrt{\frac{135^2 76^2 (211)}{59^4}} = 42.81 \end{aligned}$$

# Closed Population with Constant Effort

## **2. Multiple Samples (better precision)**

**Model – closed population ( $N$ ) except for removals**

**- constant  $p$  all animals, all samples**

**Data  $n_1, n_2, \dots, n_k$  removals**

**Approaches- Regression Approach First, Then Use of  $M_b$  and  $M_{bh}$ .**

Time	Popn. Size	Observed removals	Expected removals
1	$N$	$n_1$	$Np$
2	$N-n_1$	$n_2$	$(N-n_1)p$
3	$N-n_1-n_2$	$n_3$	$(N-n_1-n_2)p$
.	.	.	.
.	.	.	.
$i$	$N-x_i$	$n_i$	$(N-x_i)p$

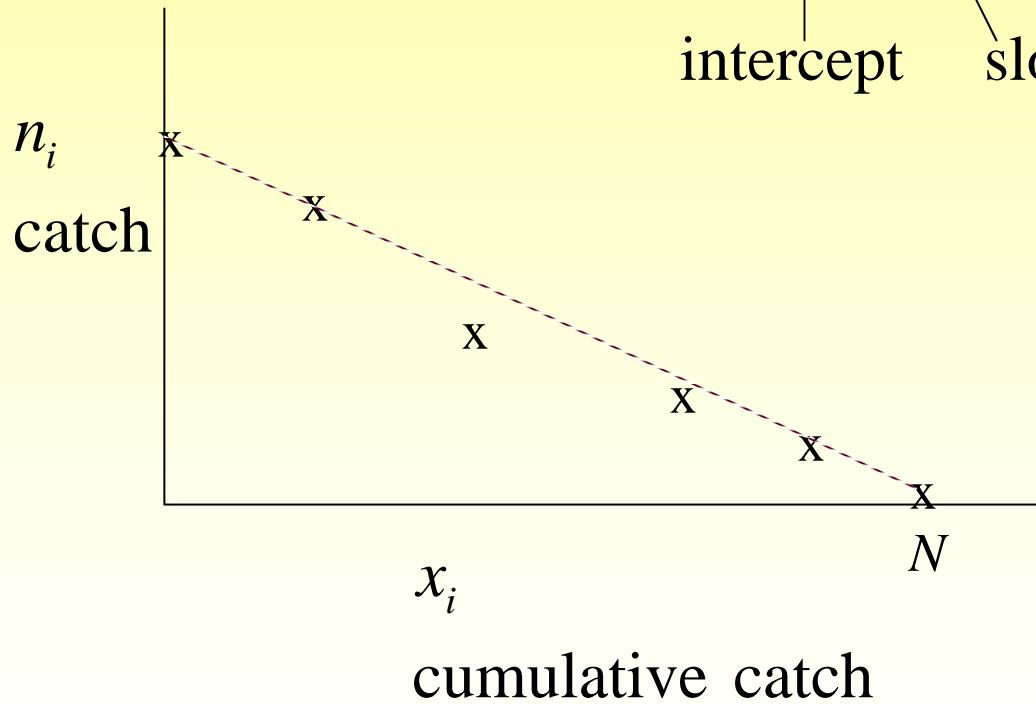
$x_i =$  cumulative catch       $n_i =$  catch

# Estimation

## (a) Regression Model

$$E(n_i) = (Np) - px_i$$

intercept      slope



# The Removal Method and Capture-Recapture

- Recall that Under Model  $M_b$ , for closed capture-recapture that animals do not contribute any information for population size estimation after first capture.
- They can be thought of as having been “removed” from the unmarked population. Thus this model is statistically equivalent to the removal model where animals are actually physically removed.

# Estimation

## (b) Use Program CAPTURE

### ( $M_b$ ) – Maximum Likelihood

Removal by marking is statistically equivalent to removal physically.

Could use Model  $M_b$ .

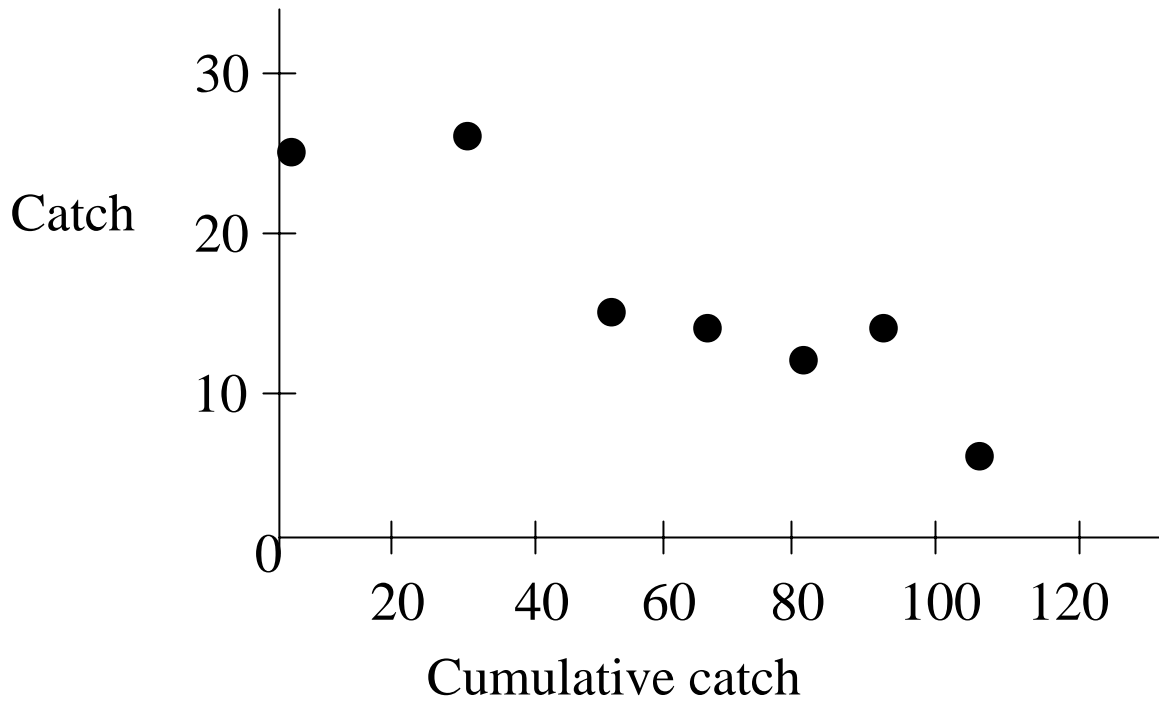
### CAPTURE ( $M_{bh}$ ) – Maximum Likelihood

Note: One can extend this removal model to  $M_{bh}$ , that is, there is heterogeneity of capture probabilities over animals. However, each animal has constant capture probability over samples.

Example: *Whitefish (Coregonus clupeaformis)*: Ricker (1958: 150)

A small lake on an island in Lake Nipigon, Ontario was fished by gill nets in an identical manner for 7 successive weeks; the same sizes of nets, positions, and lengths of sets were repeated each week. For Whitefish of fork length 13-14 inches (33-35 cm) the weekly catches and cumulative catches are given in the table below.

Week (i)	1	2	3	4	5	6	7
Catch ( $n_i$ )	25	26	15	13	12	13	5
Cumulative catch ( $x_i$ )	0	25	51	66	79	91	104



Plot of catch ( $n_i$ ) versus cumulative catch ( $x_i$ ) for a population of Whitefish: data from Ricker (1958).

# WHITE FISH -Physical Removal

Ricker (1958) p. 150

	1	2	3	4	5	6	7
Catch	25	26	15	13	12	13	5
Cumulative catch	0	25	51	66	79	91	104

Number removed = 109  
Regression Method

$$\hat{N} = 142$$

$$\hat{p} = 0.19$$

# WHITE FISH -Physical Removal

Ricker (1958) p. 150

	1	2	3	4	5	6	7
Catch	25	26	15	13	12	13	5
Cumulative catch	0	25	51	66	79	91	104

Number removed = 109 **See attached output for details**

$$M_b \quad \hat{N} = 138 \quad SE(\hat{N}) = 14.7$$

$$M_{bh} \quad \hat{N} = 138 \quad SE(\hat{N}) = 14.7$$

NO HETEROGENEITY DETECTED IN THIS EXAMPLE

# USE of CAPTURE on the Web

Example of the Removal Models (  $M(b)$ ,  $M(bh)$  )

task read population zippen

7, 'Whitefish data'

25,26,15,13,12,13,5

Task population estimate zippen

Line 2 -7-is the no. of occasions

Line 3 - Entries are the no. of removals on each occasion

For model with heterogeneity  $M(bh)$  replace zippen with removal

# USE of CAPTURE on the Web

## Whitefish Output M(b)

Occasion  $j=$  1 2 3 4 5 6 7

Total caught  $M(j)=$  0 25 51 66 79 91 104 109

Newly caught  $u(j)=$  25 26 15 13 12 13 5

Estimated probability of capture,  $\hat{p} = 0.197941$

No recaptures, so probability of recapture cannot be estimated.

Population estimate is 138 with standard error 14.6912

Approximate 95 percent confidence interval 121 to 183

Profile likelihood interval 119 to 191

# USE of CAPTURE on the Web

## Whitefish Output M(bh)

Occasion j= 1 2 3 4 5 6 7

Total caught  $M(j)$ = 0 25 51 66 79 91 104 109

Newly caught  $u(j)$ = 25 26 15 13 12 13 5

Details omitted here

Population estimate is 138 with standard error 14.6897

Approximate 95 percent confidence interval 121 to 183

Profile likelihood interval 119 to 191

In this case there is no effect of using a heterogeneity model. This is not always the case.

# Summary Closed Capture-Recapture Analysis and Use of Programs

Usually best to Use MARK!!

1. **Use CAPTURE (from MARK).** Old program but allows heterogeneity models to be fit in one analysis. *Does not use AIC and does not do multiple groups.*
2. **Use MARK closed options directly-** good *interface*, *AIC*, *multiple groups*
  - **Standard Closed Captures** –the non heterogeneity models
  - **Huggins model** –*covariates* approach to fitting heterogeneity models (not shown in class but related to logistic regression)
  - **Norris and Pollock, Pledger** –*finite mixture models* for heterogeneity modelling.

# Summary-Next Lecture

# Summary Closed Capture-Recapture Design Issues

## Precision Issues

- Need adequate capture probabilities and numbers of samples to estimate standard errors that are small enough (ie. RSE ~ 20% is good level to strive for).
- Look at Tables in Otis et al. (1978). Note that good model selection requires much larger capture probs than just estimation under one (assumed correct) model.
- Full Simulation Study-ideal
- Simpler Approximation: Use Expected Values for guesses of what the data might be like and do analysis on that data using MARK or CAPTURE. The precision (SEs) you get is a fairly good estimate as to what you would get if you ran real data with those parameter values.

# Use of Expected Value Method for Precision Evaluation

- Example  $N=500$  Lincoln Petersen Study
- Equal capture probability case  $N=500$ ,  $p=0.5$  for both periods then  $n_1=250$ ,  $n_2=250$ ,  $m_2=125$ .

$M(t)$  Estimate = 498.5 SE = 22.2

RSE = 4%

# Summary Closed Capture-Recapture Design Issues

## Minimise Model Bias- Satisfy Assumptions

1. **Closure**- Short studies, no mortality, no recruitment, no immigration or emigration.  
Check with telemetry sometimes?

## 2. Equal Catchability

**Heterogeneity**-often hard to avoid unless one can use different methods of capture in each sample which is not usually feasible. Rerandomise trap locations each time?

Collect covariate data for Huggins method or to stratify on.

**Trap Response**- often hard to avoid unless one can use different methods of capture in each sample which is not usually feasible.

**Time Variation**-try to eliminate so that simpler models can be used.

3. **No Tag Loss** – Obviously avoid, check out in pilot studies. Use double tagging method to estimate tag loss if it is a problem.

# Use of Expected Value Method for Model Bias Evaluation

- Example  $N=500$  Lincoln Petersen Study where there is heterogeneity with two groups. Average  $p=0.5$ .
  - Group 1 250 animals  $p = 0.9$
  - Group 2 250 animals  $p = 0.1$ .
  - $n_1=250, n_2=250, m_2=205$
  - Estimate = 304 Bias = -196
- Equal capture probability case  $N=500, p=0.5$  then  $n_1=250, n_2=250, m_2=125$ .
  - Estimate = 498 Bias  $\sim 0$ .

# Use of Expected Value Method for Model Bias Evaluation

Effect of Heterogeneity on LP Estimator (2 Groups  
N=500)

$p = 0.9, 0.1$  Bias = -196

$p = 0.8, 0.2$  Bias = -133

$p = 0.7, 0.3$  Bias = -70

$p = 0.6, 0.4$  Bias = -21

$p = 0.5, 0.5$  Bias = 0

Note-On exam I want you to look at trap response induced bias.