ST 506 2008 Some Brief Summary Notes on Estimation

The Two Sample Removal Model - An Example of a Sampling and Estimation Problem

As an example of a statistical sampling model let us consider a study to monitor freshwater fish in streams, we want to estimate population size and detection probability using an electro-fishing removal study.

We denote the data collected:
n_1 fish captured and removed in sample 1
n_2 fish captured and removed in sample 2

We *assume*:
a closed population and
equal detection probability of all fish at both time points.

The parameters are N the population size and p the probability of detection at each time.

Note that the assumption of equal catchability is stronger than in the simple capture-recapture model we consider a bit later in the semester because we require equal capture probabilities for the two samples.)

$$n_1 / N \cong n_2 / (N - n_1)$$

$$\hat{N} = n_1^2 / (n_1 - n_2)$$

$$\hat{p} = (n_1 - n_2) / n_1$$

$$\hat{N} = n_1 / \hat{p}$$

We use the "hat " symbol to emphasize that this is an estimate rather than the true parameter. This is a classic example of the sort of model based estimator we will be using in this class. Violation of model assumptions can lead to serious biases in the estimator. We also need to be concerned about the sample sizes as they will determine the precision of the estimator. The twin themes of model bias and precision will appear repeatedly in what we do in this class.

Properties of Estimators

Bias

The difference between the expected value and the parameter.

Precision (Variance and St Error)

The average squared deviation from the expected value of the estimator.

Accuracy (Mean Squared Error = Variance+ Bias^2)

The average squared deviation from the parameter.

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Methods of Estimation of Parameters

Method of Moments

We basically used this method with the removal model described earlier when I was describing the intuitive derivation. One expresses what the expected value of some observed value would be equal to in terms of the unknown population parameters.

Method of Least Squares

You should have seen this in a regression class such as ST 511-512. Consider fitting a straight line to some data. You could minimise the sum of squared differences between the fitted line and the observed points.

$$Minimise \sum (y_i - \alpha - \beta x_i)^2$$

The resulting estimators of the slope (β) and intercept (α) are well known and called the least squares estimators. The can also be shown to be maximum likelihood if the errors are normally distributed.

Method of Maximum Likelihood

Derive the probability distribution of the observed data as a function of the parameters and then view this as a function of the parameters, this is the likelihood function. Find the values of the parameters which maximise this function. These are the maximum likelihood estimators (MLEs).

A very simple example

R Retained tag L Lost tag

p - prob tag retained, (1 - p) - prob tag lost

$$\mathrm{P(RRLRLRRRRL)} = \mathrm{p}^7 (1 - \mathrm{p})^3$$

$$\mathrm{L(p)} = \mathrm{p}^7 (1 - \mathrm{p})^3$$

The likelihood is maximised at

$$\hat{\mathrm{p}} = \mathrm{x/n} = 7/10 = 0.7$$

The shape of the likelihood will be shown on the whiteboard.

Maximum likelihood estimators first developed by R. A Fisher have very good large sample properties compared to other estimators. They are consistent, have no large sample bias if the model is correct, and have the smallest large sample variance. This together with other examples is discussed at greater length in the text and in later lectures. Most likelihoods will have many parameters and MLEs will have to be computed using software like MARK.