ST 432 Homework Set 9
Due Thursday April 2, 2009

1. Q1 P169 Double Sampling Question using the ratio estimator

ST 432 Homework Set 10
Due Tuesday April 7, 2009

1. I would also like you to answer a question about the maximum likelihood estimator of N in a Lincoln-Petersen capture-recapture model with two samples. The likelihood is

$$L = P(n_1)P(m_2 \text{ given } n_1)P((n_2 - m_2)\text{given } n_1)$$

$$L = \frac{N!}{n_1!(N-n_1)!}\, p_1^{n_1}(1-p_1)^{N-n_1}\, \frac{n_1!}{m_2!(n_1-m_2)!}\, p_2^{m_2}(1-p_2)^{n_1-m_2}$$

$$\frac{(N-n_1)!}{(n_2-m_2)!(N-n_1-n_2+m_2)!}\, p_2^{n_2-m_2}(1-p_2)^{N-n_1-n_2+m_2}$$

This simplifies to the multinomial with 3 parameters N, p_1, p_2

$$L = \frac{N!}{m_2!(n_1-m_2)!(n_2-m_2)!(N-n_1-n_2+m_2)!}$$

$$[p_1 p_2]^{m_2}[p_1(1-p_2)]^{n_1-m_2}[(1-p_1)p_2]^{n_2-m_2}[(1-p_1)(1-p_2)]^{N-n_1-n_2+m_2}$$

I would like you to show that the MLES are

$$\hat{N} = n_1 n_2 / m_2$$

$$\hat{p}_1 = m_2 / n_2$$

$$\hat{p}_2 = m_2 / n_1$$

The usual approach is to take the log of the likelihood, and take partial derivatives with respect to each parameter and set equal to 0. Here use that for p_1 and p_2 but take L(N)/L(N - 1) = 1 as the third equation. Please use L not logL in this 3rd equation. Also do not try to differentiate logL with respect to N! That would be an exercise in frustration.

Note - I will talk about this a little in next Tuesdays class to help get you started.