ST512
Fall Semester, 2006
Quiz 2 solutions

1. True or false: circle one (24 pts total)

TRUE	(i) In an analysis with multiple comparisons, the familywise error rate is the probability of at least one type I error
TRUE	(ii) In an analysis with multiple comparisons, let $S > 0$ denote the number of tests for which the null hypotheses are rejected and FP denote the number that are falsely rejected, then $E(FP/S)$ is called the false discovery rate (FDR)
FALSE	(iii) For tests derived from multiple confidence intervals with simultaneous confidence level .95, the probability of at most one type I error is at least .95.
TRUE	(iv) In making all pairwise comparisons among t means in a one-factor experiment, the Bonferroni procedure ($k = t(t-1)/2$) will lead to more expected type II errors than Tukey's procedure.
TRUE	(v) A design in which an equal number of units is randomized to each treatment is called *balanced*
	For the following three problems, consider a completely randomized design, with normally distributed errors with equal variances
FALSE	(vi) For a given H_0 and H_1, the sample size required to achieve a type II error rate of $\beta = 0.1$ is greater than that required to achieve $\beta = 0.05$.
TRUE	(vii) The sample size required to achieve a type II error rate of $\beta = 0.1$ is greater if the population variances are $\sigma = 2$ than if $\sigma = 1$.
TRUE	(viii) For fixed numerator and denominator degrees of freedom ν_1, ν_2, the cumulative distribution function of the non-central F-distribution evaluated at the fixed critical value $F(\alpha, \nu_1, \nu_2)$ decreases as the noncentrality parameter γ increases.

2. (20 pts) The Peace Corps conducts an experiment to assess four spanish instruction techniques. A language aptitude test (z) is administered to $N = 40$ volunteers who are then randomized to $t = 4$ groups. They work abroad for two years and give a rating (y) of the usefulness of their program on their return (10 point scale). Consider the model

$$E(Y|X_L, X_A, Z) = \beta_0 + \beta_1 x_L + \beta_2 x_A + \beta_3 x_L x_A + \beta(z - \bar{z}).$$

Techniques, indicator variables and SAS code and output for a regression analysis are given below. Questions appear on the following page.

Technique	x_L	x_A	$x_L x_A$	Adj. Mean
Lecture with Audiotape	1	1		
Lecture without Audiotape	1	0		
Conversation with Audiotape	0	1		(SE =)
Conversation without Audiotape	0	0		(SE =)

```
proc reg data=volunteers;
   model score = zdiff xL xA xLA /covb;     /* zdiff=z-zbar */
run;
```

```
The SAS System
The REG Procedure
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Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	123.11696	30.77924	4.73	0.0051
Error	27	175.75804	6.50956		
Corrected Total	31	298.87500			

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Type I SS	Type II SS
Intercept	1	3.48488	0.90570	3.85	0.0007	1035.12500	96.37450
zdiff	1	0.21475	0.07422	2.89	0.0075	62.09930	54.49196
xL	1	2.53445	1.29192	1.96	0.0602	2.59775	25.05233
xA	1	3.82131	1.27583	3.00	0.0058	28.11916	58.39739
xLA	1	-3.90104	1.80813	-2.16	0.0400	30.30075	30.30075

Covariance of Estimates

Variable	Intercept	zdiff	xL	xA	xLA
Intercept	0.82	0.01	-0.83	-0.82	0.82
zdiff	0.01	0.01	-0.02	-0.00	0.01
xL	-0.83	-0.02	1.67	0.82	-1.65
xA	-0.82	-0.00	0.82	1.63	-1.63
xLA	0.82	0.01	-1.65	-1.63	3.27

(a) Report the extra sum of squares for technique before and after adjustment for the language aptitude test score.

$$\begin{aligned} R(\beta_1, \beta_2, \beta_3|\beta_0) &= R(\beta_1, \beta_2, \beta_3, \beta|\beta_0) - R(\beta|\beta_1, \beta_2, \beta_3, \beta_0) \\ &= 123.1 - 54.5 \\ R(\beta_1, \beta_2, \beta_3|\beta_0, \beta) &= R(\beta_1|\beta_0, \beta) + R(\beta_2|\beta_0, \beta, \beta_1) + R(\beta_3|\beta_0, \beta, \beta_1, \beta_2) \\ &= 2.6 + 28.1 + 30.3 \end{aligned}$$

(b) Report the adjusted mean rating for each technique. (use prior table if you like)

$$\begin{array}{lccccccccccc} \text{L+A:} & \hat\beta_0 & + & \hat\beta_1 & + & \hat\beta_2 & + & \hat\beta_3 & + & \beta(0) & = & 3.5 + 2.5 + 3.8 - 3.9 \\ \text{L-A:} & \hat\beta_0 & + & \hat\beta_1 & & & & & + & \beta(0) & = & 3.5 + 2.5 \\ \text{C+A:} & \hat\beta_0 & + & & & \hat\beta_2 & & & + & \beta(0) & = & 3.5 + 3.8 \\ \text{C-A:} & \hat\beta_0 & + & & & & & & & \beta(0) & = & 3.5 \end{array}$$

(c) Report the standard error of the adjusted means for the two 'conversation' techniques. (use prior table if you like) (Let $\hat\beta$ denote the vector of regression coefficients):

$$\begin{aligned} SE(\hat\beta_0 + \hat\beta_2) &= \sqrt{(1,0,1,0,0)\text{Cov}(\hat\beta)(1,0,1,0,0)^T} \\ &= \sqrt{.82 + 1.67 + 2(-.83)} \\ &= 0.91 \end{aligned}$$

(d) Estimate the simple effects of lecture type with and without audio (after adjustment for language aptitude z).

$$\begin{aligned} \text{with audio:} \hat\beta_1 + \hat\beta_3 &= 2.5 + (-3.9) \\ \text{without audio:} \hat\beta_1 &= 2.5 \end{aligned}$$

(e) Report the difference of the two simple effects in part (d) along with a p-value for a test of no mean difference in the population.

$$\hat\beta_3 = -3.9 (SE = 1.8)$$

3. (18 pts) Consider a one-factor weed-growth experiment in which $t = 6$ different herbicide treatments are randomized to 24 similar weed plants.

Treatment number	Herbicide	sample size	Prot. LSD grouping	Tukey grouping	sample mean
1	Control	4	A	A	10
2	Ordram 8	4	B	B	7.5
3	Ordram 8-mp	4	BC	B	7.0
4	Ordram 8-hip	4	C	B	5.9
5	RedRaider 8	4	D	C	3.1
6	RedRaider 8-hip	4	D	C	2.5

Suppose that the researcher would like to make some number $k \leq \binom{6}{2} = 30$ pairwise comparisons, but not all of them. He also wants to control the familywise error rate at $\alpha = 0.05$.

(a) What is the largest number of comparisons k he can make using the Bonferroni procedure and retain more power than the Tukey procedure? $k = 8$?, $k = 9$?
Upon inspection using tables C.10 and C.5,

$$t(\frac{.025}{9}, 18)\sqrt{MSE\frac{2}{n}} < q(.05, 6, 18)\sqrt{MSE\frac{1}{n}} < t(\frac{.025}{10}, 18)\sqrt{MSE\frac{2}{n}}$$

so that $k = 9$ is the largest number of comparisons where Bonferroni wins.

(b) Suppose $MS[E] = 1$ is observed along with the six sample treatment means:

$$\bar{y}_{1+} = 10, \bar{y}_{2+} = 7.5, \bar{y}_{3+} = 7.0, \bar{y}_{4+} = 5.9, \bar{y}_{5+} = 3.1, \bar{y}_{6+} = 2.5$$

To make all pairwise comparisons, while controlling the FWE at .05, obtain the LSD for Fisher's protected procedure and also Tukey's HSD. Use common letters in the rightmost columns in the table above to indicate means that do not differ significantly. (The overall F-test is significant at .05)

$$\begin{aligned} LSD &= t(.025, 18)\sqrt{MSE\tfrac{2}{4}} &= 1.48 \\ HSD &= q(.05, 6, 18)\sqrt{MSE\tfrac{1}{4}} &= 2.25 \end{aligned}$$

(c) The *mp* and *hip* indicate added pressures with which the herbicides are applied. Suppose these added pressure have no real effect. Which procedure led to a type I error and for which pairwise comparison? This means that treatment means 2,3 and 4 are equal to each other and so are 5 and 6, but according to LSD, treatments 2 and 4 differ significantly.

4. (20 pts) A food scientist measures myosin degradation rate (y) in tuna at various cooking temperatures. Single samples of belly meat are taken from 9 fish, and randomized to three temperature treatments, $n = 3$ each. Samples of tail meat are taken from another 9 fish and randomized to temperatures, with the following results:

Position	Belly meat			Tail meat		
Temperature	50^o	60^o	70^o	50^o	60^o	70^o
Treatment (i)	1	2	3	4	5	6
Treatment mean ($\bar{y}_i$)	35	17	10	30	14	6
Treatment variance (s_i^2)	4	4	4	16	16	4

Assume errors for individual fish are normally distributed. Consider the five contrasts below. Use $\alpha = .05$ and $F(.05, 1, 12) = 4.75$ and $F(.05, 2, 12) = 3.89$. Recall also that coefficients for orthogonal polynomial contrasts with three equally spaced levels are $(-1, 0, 1)$ for linear and $(1, -2, 1)$ for quadratic.

$$
\begin{array}{ll|l}
\hat{\theta}_1 = \bar{y}_1 + \bar{y}_2 + \bar{y}_3 - \bar{y}_4 - \bar{y}_5 - \bar{y}_6 & = +12 & \\
\hat{\theta}_2 = -\bar{y}_1 + \bar{y}_3 - \bar{y}_4 + \bar{y}_6 & = -49 & \\
\hat{\theta}_3 = \bar{y}_1 - 2\bar{y}_2 + \bar{y}_3 + \bar{y}_4 - 2\bar{y}_5 + \bar{y}_6 & = 19 & \\
\hat{\theta}_4 = -\bar{y}_1 + \bar{y}_3 + \bar{y}_4 - \bar{y}_6 & = -1 & SS(\theta_4) = 0.75 \\
\hat{\theta}_5 = \bar{y}_1 - 2\bar{y}_2 + \bar{y}_3 - \bar{y}_4 + 2\bar{y}_5 - \bar{y}_6 & = 3 & SS(\theta_5) = 2.25
\end{array}
$$

(a) What two factors are under study in this experiment? position and temperature

(b) Obtain $MS[E]$. $MSE = \frac{1}{6}(4 + 4 + 4 + 16 + 16 + 4) = 8$

(c) What effect is being estimated by $\hat{\theta}_1$? Obtain $SS(\hat{\theta}_1)$. Is the effect significant?
main effect of position:

$$SS(\theta_1) = \frac{\hat{\theta}_1^2}{\frac{6}{3}} = 72, F = \frac{72}{8} \text{(sig)}$$

(d) What effect is being estimated by $\hat{\theta}_2$? Obtain $SS(\hat{\theta}_2)$. Is the effect significant?
linear effect of temperature (averaged over position):

$$SS(\theta_2) = \frac{\hat{\theta}_2^2}{\frac{4}{3}} = 1801, F = \frac{1801}{8} \text{(sig)}$$

(e) What effect is being estimated by $\hat{\theta}_3$? Obtain $SS(\hat{\theta}_3)$. Is the effect significant? quadratic effect of temperature (averaged over position):

$$SS(\theta_3) = \frac{\hat{\theta}_2^2}{\frac{12}{3}} = 90, F = \frac{90}{8}(\text{sig})$$

(f) Find the variance of $\hat{\theta}_1 + \hat{\theta}_2$. $\text{Var}(\hat{\theta}_1 + \hat{\theta}_2) = \sigma^2(\frac{6}{3} + \frac{4}{3})$

(g) Find the interaction sum of squares, $SS(\text{temp} \times \text{position})$ on $df = 2$. (Note that θ_4 and θ_5 quantify interaction and $SS[\text{temp}] = SS(\theta_2) + SS(\theta_3)$ so that main effects sums of squares have already been obtained.) $SS(\theta_4) + SS(\theta_5) = 3$

(h) TRUE : these five contrasts are orthogonal.

(i) Obtain the model sum of squares for the full factorial effects model on $df = 5$.

$$SS(Trt) = 72(\text{pos}) + 1891(\text{temp}) + 3(\text{interaction})\text{on } df = 1 + 2 + 2 = 5.$$

(j) Briefly characterize the effects of position and temperature on myosin degradation. (Sketch an interaction plot if you like.) As temperature increases, degradation rates decrease non-linearly. This decrease is constant across meat positions, with belly degrading faster across all temps.

5. (18 pts) A biomedical engineer randomizes $N = 24$ tissue collagen constructs seeded with stem cells to 8 experimental conditions (combinations of medium (A) seeding density (B), and strain (C). He then measures recovery scores (y) after impaction treatment, as listed in the tables of means below. Questions appear on the next page.

		Medium=B		Medium=O	
		Strain		Strain	
		10	12	10	12
Density	30	609	780	642	493
	60	479	792	538	463

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                         The GLM Procedure
                      Class Level Information

                Class          Levels    Values
                medium              2    B O
                density             2    30 60
                strain              2    10 12

                                    Sum of
Source                     DF      Squares     Mean Square    F Value    Pr > F
Model                       7  359430.0000      51347.1429      43.15    <.0001
Error                      16   19040.0000       1190.0000
Corrected Total            23  378470.0000

Source                     DF  Type III SS     Mean Square    F Value    Pr > F
medium                      1  102966.0000     102966.0000      86.53    <.0001
density                     1   23814.0000      23814.0000      20.01    0.0004
medium*density              1      96.0000         96.0000       0.08    0.7800
strain                      1   25350.0000      25350.0000      21.30    0.0003
medium*strain               1  187974.0000     187974.0000     157.96    <.0001
density*strain              1   17496.0000      17496.0000      14.70    0.0015
medium*densit*strain        1    1734.0000       1734.0000       1.46    0.2449

                       Least Squares Means

                   medium          y LSMEAN
                   B             665.000000
                   O             534.000000

             medium     density          y LSMEAN
             B          30             694.500000
             B          60             635.500000
             O          30             567.500000
             O          60             500.500000
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(a) Report the F-ratio and p-value for a test that the population mean compaction score is constant across all 8 experimental conditions.

$$F = 43.5, p < 0.0001$$

(b) Placing strain on the horizontal axis, sketch an interaction plot, or possibly two interaction plots.

(c) Estimate the simple strain effect at each combination of medium and density.

$$\begin{aligned} \hat{\mu}(CA_1B_1) &= 770 - 609 \\ \hat{\mu}(CA_1B_2) &= 792 - 479 \\ \hat{\mu}(CA_2B_1) &= 493 - 642 \\ \hat{\mu}(CA_2B_2) &= 463 - 538 \end{aligned}$$

(d) Report the standard error for any of the estimated effects in (c).

$$\sqrt{\frac{2}{3} MS[E]} = 28.2$$

(e) Estimate the strain by medium interaction for density=30

$$\hat{\mu}[ACB_1] = 780 - 609 - (493 - 642) = 320$$

(division by 2 ok)

(f) Report a p-value for a test that the interaction estimated in part (e) is the same at that estimated at density=60. Is it plausible that the apparent qualitative interaction between strain and medium is constant across density? Yes, plausible since a test of no interaction has p-value 0.2449 (from output)