ST512-R
Homework #7

1. (Taken from Rao 16.8, p. 765) An experiment evaluates the taste of three brands of ice cream under different storage conditions. The expt. calls for 12 coolers to be randomized to the 3 brands (4 coolers per brand). Within each cooler, there is room for three half-gallon cartons of ice cream, all from the same brand. One carton was sampled at random at the end of one day, then at the end of one week and then one month. Each carton was evaluated for flavor by a panel of tasters. These panel scores can be found in "icecream.dat" (or also "storage.dat") on the course webpage.

   (a) This is a split-plot experiment. What are the whole plot units?

   (b) What is/are the whole plot factor(s)?

   (c) What is/are the sub plot factor(s)?

   (d) Test for an interaction between storage time and brand. (Use $\alpha = 0.05$.)

   (e) Test for main effects of storage time. Carry out pairwise comparisons among the three storage time tastes, using a Bonferroni correction to control the exptwise error rate at .05.

   (f) Test for main effects of brand. Report the standard error on any pairwise difference among brands.

   (g) Suppose you had averaged the data over storage time, reducing the total number of observations from 36 down to 12. Would the F-ratio for brand effect in a a one-way ANOVA be the same as the F-ratio from part (g)? Try it and see.

2. Consider an experiment like the one described in #5, with a levels of a between-plots factor A, n plots per level of A, for a total of nA plots and b levels of a within-plots factor B. Let Y_{ijk} denote the measurement at level j of factor B, from the k^{th} whole plot randomized to level i of factor A. The model is then

$$Y_{ijk} = \mu + \alpha_i + S_{k(i)} + \beta_j + (\alpha\beta)_{ij} + E_{ijk}$$

   where $S_{k(i)} \overset{iid}{\sim} N(0, \sigma_s^2)$ and $E_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$, independently of $S_{k(i)}$.

   (a) Derive the standard error for a difference of means across levels of ...

      i. B, for a fixed level of A, for example, $\bar{Y}_{12+} - \bar{Y}_{11+}$.
      ii. B, after averaging over levels of A, for example, $\bar{Y}_{+2+} - \bar{Y}_{+1+}$
      iii. A, for a fixed level of B, for example, $\bar{Y}_{21+} - \bar{Y}_{11+}$.
      iv. A, after averaging over levels of B, for example, $\bar{Y}_{2++} - \bar{Y}_{1++}$
      v. A, and across levels of B, for example, $\bar{Y}_{22+} - \bar{Y}_{11+}$

   (b) For each of the above, construct an estimate of the standard error using linear combinations of MS terms and indicate whether or not an approximation for df is required.

3. (Rao, (1998) eg 16.5) An experiment investigates three formulations of a diet for rats. The response is absorption of a certain chemical by the kidneys. The design involves four litters, each with three rats (a total of 12 rats). Within each litter, rats are randomized to diet formulation. Another factor of interest is the method of measuring this absorption. There are three methods up for investigation, with differences that may be subtle compared to diet formulation effects. So, three specimens are sampled from *each* rat, and these specimens are randomized to the methods. Data are available as "absorb.dat"

   (a) Identify the name of the experimental design used here.

   (b) Propose a statistical model.

   (c) Sketch an ANOVA table, with columns for source, *df* and EMS.

   (d) Test for an interaction between formulation and method. Use $\alpha = 0.05$.

   (e) Test ($\alpha = 0.05$) for simple method effects separately at each level of formulation.

   (f) Test for the simple effects of formulation using method 3. Carry out all three pairwise comparisons among formulations using this method, identifying any significant differences.

4. A food scientist studies the effects of three factors on the viscosity of ice cream:

   - A: temperature (2 levels)
   - B: pressure (2 levels)
   - C: recipe (3 levels).

   The production equipment is such that temperature and pressure must be held constant for the entire day. The experiment is conducted over 8 days, which are randomly assigned to the four treatment combinations of temperature and pressure. Three batches are produced each day, one of each recipe, for a total of 24 batches. Each batch will have a single viscosity measurement taken. Pertinent SAS code and output are given on the next page.

   (a) Propose a mixed model for the viscosity measurements with random day effects. Take the effects of all three treatment factors, A, B, C and their first and second order interactions to be fixed. Assume normal distributions for all random effects.

   (b) Carry out F-tests for the four hypotheses listed below. For each, report the F-ratio, the associated degrees of freedom, a conclusion regarding the hypothesis, and where available from the output, a p-value.

Hypothesis
No 2^{nd} order interaction between temperature, pressure and recipe
No 1^{st} order interaction between temperature and pressure
No main effect of pressure
No main effect of recipe

   (c) Estimate the variance component for the random day effect.

   (d) Report an estimate and standard error for

      i. the marginal mean of recipe 1
      ii. the difference between recipes 1 and 2
      iii. the marginal mean of temperature 1
      iv. the difference between temperatures 1 and 2

```
proc glm;
   class temp pressure day recipe;
   model viscosity=temp|pressure|recipe day(temp*pressure);
   lsmeans temp|pressure|recipe;
run;
```

The SAS System
The GLM Procedure

Class	Levels	Values
temp	2	high low
pressure	2	high low
day	8	1 2 3 4 5 6 7 8
recipe	3	a b c

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	15	54.90291667	3.66019444	14.17	0.0004
Error	8	2.06666667	0.25833333		
Corrected Total	23	56.96958333			

R-Square	Coeff Var	Root MSE	viscosity Mean
0.963723	9.957845	0.508265	5.104167

Source	DF	Type III SS	Mean Square	F Value	Pr > F
temp	1	9.75375000	9.75375000	37.76	0.0003
pressure	1	26.67041667	26.67041667	103.24	<.0001
temp*pressure	1	1.00041667	1.00041667	3.87	0.0846
recipe	2	4.30583333	2.15291667	8.33	0.0111
temp*recipe	2	0.36750000	0.18375000	0.71	0.5196
pressure*recipe	2	0.52583333	0.26291667	1.02	0.4038
temp*pressure*recipe	2	0.26083333	0.13041667	0.50	0.6216
day(temp*pressure)	4	12.01833333	3.00458333	11.63	0.0020

Level of temp	Level of pressure	Level of recipe	N	viscosity Mean	viscosity Std Dev
high	high	a	2	5.50000000	2.82842712
high	high	b	2	5.10000000	1.27279221
high	high	c	2	5.35000000	1.62634560
high	low	a	2	3.70000000	0.28284271
high	low	b	2	2.85000000	0.21213203
high	low	c	2	4.30000000	0.56568542
low	high	a	2	6.80000000	0.14142136
low	high	b	2	6.55000000	0.91923882
low	high	c	2	7.65000000	0.35355339
low	low	a	2	4.40000000	0.28284271
low	low	b	2	3.85000000	0.35355339
low	low	c	2	5.20000000	0.42426407