ST 512 Homework assignment #5, Fall, 2006, Osborne Solutions

1. (Taken from Rao 16.8, p. 765)

   (a) Coolers

   (b) Brands

   (c) Storage time

   (d)
$$F_{AB} = \frac{MS(AB)}{MS(E} = 0.19, p = 0.9379 \text{ on } df = 4, 18$$

   (e)
$$F_B = \frac{MS(B)}{MS(E)} = 10.3, p = 0.0010 \text{ on } df = 2, 18$$

$$\begin{aligned} \widehat{SE}(\bar{y}_{+j_1+} - \bar{y}_{+j_2+}) &= \sqrt{\frac{2}{12} MS(E)} \\ &= 2.048(\text{ on } df = 18 \end{aligned}$$

   The minimum significant difference is $t(.025/3, 18) * SE = 5.4$. The only pairwise difference not exceeding MSD is the one comparing times 2 and 3.

   (f) pairwise comparison among brands.
$$F_A = \frac{MS(A)}{MS(cooler(A))} = 0.31, p = 0.7411 \text{ on } df = 2, 9$$

$$\begin{aligned} \widehat{SE}(\bar{y}_{i_1++} - \bar{y}_{i_2++}) &= \sqrt{\frac{2}{12} MS(cooler(A))} \\ &= 3.42(\text{ on } df = 9 \end{aligned}$$

   (g) F-ratio is 0.74 on $df = 2, 9$ either way.

2. (a) i. B, for a fixed level of A, for example, $\bar{Y}_{12+} - \bar{Y}_{11+}$

$$\begin{aligned} \bar{Y}_{12+} - \bar{Y}_{11+} &= \beta_2 - \beta_1 + (\alpha\beta)_{12} - (\alpha\beta)_{11} + \bar{S}_{+(1)} - \bar{S}_{+(1)} + \bar{E}_{12+} - \bar{E}_{11+} \\ SE &= \sqrt{\frac{2}{n}\sigma^2} \\ \widehat{SE} &= \sqrt{\frac{2}{n} MS(E)} \text{ on } df = (b-1)(n-1)a \end{aligned}$$

ii. B, after averaging over levels of A, for example, $\bar{Y}_{+2+} - \bar{Y}_{+1+}$

$$\begin{aligned}
\bar{Y}_{+2+} - \bar{Y}_{+1+} &= \beta_2 - \beta_1 + \bar{E}_{+2+} - \bar{E}_{+1+} \\
SE &= \sqrt{\frac{2}{na}\sigma^2} \\
\widehat{SE} &= \sqrt{\frac{2}{na}MS(E)} \text{ on } df = (b-1)(n-1)a
\end{aligned}$$

iii. A, for a fixed level of B, for example, $\bar{Y}_{21+} - \bar{Y}_{11+}$

$$\begin{aligned}
\bar{Y}_{21+} - \bar{Y}_{11+} &= \alpha_2 - \alpha_1 + (\alpha\beta)_{21} - (\alpha\beta)_{11} + +\bar{S}_{+(2)} - \bar{S}_{+(1)} + \bar{E}_{21+} - \bar{E}_{11+} \\
SE &= \sqrt{\frac{2}{n}(\sigma_s^2 + \sigma^2)} \\
\widehat{SE} &= \sqrt{\frac{2}{n}(\hat{\sigma}_s^2 + \hat{\sigma}^2)} \\
&= \sqrt{\frac{2}{n}((MS(S(A)) - MSE)/b + MSE)} \\
&= \sqrt{\frac{2}{nb}(MS(S(A)) + (b-1)MSE)} \text{ on } \widehat{df}
\end{aligned}$$

iv. A, after averaging over levels of B, for example, $\bar{Y}_{2++} - \bar{Y}_{1++}$

$$\begin{aligned}
\bar{Y}_{2++} - \bar{Y}_{1++} &= \text{(fixed effects)} + \bar{S}_{+(2)} - \bar{S}_{+(1)} + \bar{E}_{2++} - \bar{E}_{1++} \\
SE &= \sqrt{\frac{2}{nb}(b\sigma_s^2 + \sigma^2)} \\
\widehat{SE} &= \sqrt{\frac{2}{nb}(MS(S(A))} \text{ on } df = (n-1)a
\end{aligned}$$

v. A, and across levels of B, for example, $\bar{Y}_{22+} - \bar{Y}_{11+}$

$$\begin{aligned}
\bar{Y}_{22+} - \bar{Y}_{11+} &= \text{(fixed effects)} + \bar{S}_{+(2)} - \bar{S}_{+(1)} + \bar{E}_{22+} - \bar{E}_{11+} \\
SE &= \sqrt{\frac{2}{n}(\sigma_s^2 + \sigma^2)} \\
\widehat{SE} &= \sqrt{\frac{2}{n}(\hat{\sigma}_s^2 + \hat{\sigma}^2)} \\
&= \sqrt{\frac{2}{n}((MS(S(A)) - MSE)/b + MSE)} \\
&= \sqrt{\frac{2}{nb}(MS(S(A)) + (b-1)MSE)} \text{ on } \widehat{df}
\end{aligned}$$

(b) For each of the above, construct an estimate of the standard error using linear combinations of MS terms and indicate whether or not an approximation for *df* is required. (see above)

3. Rats problem - Rao 16.6

(a) Split-plot in blocks. The arrangement of the whole plot treatment factor (diet formulation) among the rats (whole plot units) is in blocks; with litters forming the blocks.

(b)

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + L_k + (\alpha L)_{ik} + E_{ijk}$$

where $L_k \overset{iid}{\sim} N(0, \sigma_L^2), (\alpha L)_{ik} \overset{iid}{\sim} N(0, \sigma_{\alpha L}^2)$ and $E_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

(c) ANOVA table

Source	*df*	EMS
Formulation	2	$\sigma^2 + 12\psi_A^2 + 3\sigma_{\alpha L}^2$
Litter	3	$\sigma^2 + 9\sigma_L^2 + 3\sigma_{\alpha L}^2$
Formulation * Litter (aka error A)	6	$\sigma^2 + 3\sigma_{\alpha L}^2$
Method	2	$\sigma^2 + 12\psi_B^2$
Method*Formulation	4	$\sigma^2 + 4\psi_{AB}^2$
Error (aka error B)	18	σ^2

(d)

$$F_{form*method} = \frac{MS(form * method)}{MS(E)} = 43(p < 0.0001)$$

(e)

$$\begin{aligned} F_{M,F_1} &= 101.37 \\ F_{M,F_2} &= 98.6 \\ F_{M,F_3} &= 114.8 \end{aligned}$$

(All three F-ratios have $p < 0.0001$ on $df = 2, 18$.)

(f)

$$F_{F,M_3} = 55.5(p < 0.0001, df = 2, 18.8)$$

(A more explicit expression for this F-ratio is the following:

$$\begin{aligned} F &= \frac{\sum_i \sum_k (\bar{y}_{i3+} - \bar{y}_{+3+})^2/2}{\hat{\sigma}_{\alpha L}^2 + \hat{\sigma}^2} \\ &= \frac{80.8/2}{0.72} \\ &= 55.5 \end{aligned}$$

with $\widehat{df} = 18.8$ by Satterthwaite. The only non-significant difference using method three is the one comparing formulation 1 and formulation 3.)

4. (a) What units serve as blocks for the pressure settings? institutions
   (b) What are the whole plot units? days
   (c) What is the whole plot factor? pressure
   (d) What is the within plot factor? recipe
   (e) Outline an ANOVA table, providing the sources of variability and degrees of freedom from a partition of the total sum of squares on 27 degrees of freedom into 6 orthogonal components.

Source	*df*
Pressure	2
Institution	2
P*I, aka "Error A"	4
Recipe	2
P*R	4
Error aka"Error B"	