

1. Consider a multiple linear regression involving a sample of $n = 112$ NFL football games from the latter half of the 1989 season. Let y denote the margin of victory for the home team (negative if a loss) and let x_1 and x_2 denote the average margin of victory for the home team and visiting team, respectively, during the first half of the season. Consider the following regression models for the mean outcome:

- Model 1: $\mu(x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$
- Model 2: $\mu(x_1, x_2) = \beta_0 + \beta_1 x_1 - \beta_1 x_2$

SAS PROC REG was used to get the ANOVA tables and parameter estimates for these two models as given in the (abbreviated) output on the following page.

- (a) Obtain a 95% confidence interval for the sum of the effects of the average margin of victory for the home team and the visiting team. That is, obtain a 95% confidence interval for $\beta_1 + \beta_2$, using model 1. (4 points) Since $t(.025, 109) = 1.98$, the 95% confidence interval is given by

$$\hat{\beta}_1 + \hat{\beta}_2 \pm 1.98SE(\hat{\beta}_1 + \hat{\beta}_2)$$

The F -ratio for comparing models 1 and 2 is given by

$$\begin{aligned} F &= \frac{SS(R)_{\text{full}} - SS(R)_{\text{red}}}{MS(E)_{\text{full}}} \\ &= \frac{17376 - 17151}{157.96} \\ &= 1.42 \end{aligned}$$

Note that this is also the square of the t -statistic for a test of $H_0 : \beta_1 + \beta_2 = 0$:

$$\begin{aligned} F &= t^2 \\ &= \left(\frac{\hat{\beta}_1 + \hat{\beta}_2}{SE} \right)^2 \\ &= \left(\frac{1.209 - .863}{SE} \right)^2 \\ &= \left(\frac{0.346}{SE} \right)^2 \end{aligned}$$

This implies

$$SE = \hat{\beta}_1 + \hat{\beta}_2 / t = 0.346 / \sqrt{1.42} = 0.29$$

for a 95% confidence interval of

$$0.346 \pm 1.98(0.29) \text{ or } 0.346 \pm 0.575 \text{ or } (-.229, .921)$$

- (b) Use model 2 to obtain a 95% prediction interval for the amount by which the Super-Bowl champion Indianapolis Colts will win their upcoming game in which they host the undefeated New England Patriots. (1 point) A glance at the standings in the News and Observer reveals that the average margins of victory for these two teams are $x_1 =$ (the hometeam Colts) and $x_2 =$ (the road Patriots). This yields a prediction of . Place your bets!

```

proc reg data=nfl89;
  model outcome=x1 x2;
  model outcome=diff; /* diff=x1-x2 */
run;

```

1

The SAS System
 The REG Procedure
 Model: MODEL1
 Dependent Variable: outcome

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	9863.77734	4931.88867	31.34	<.0001
Error	109	17151	157.34805		
Corrected Total	111	27015			

Root MSE	12.54384	R-Square	0.3651
Dependent Mean	3.60714	Adj R-Sq	0.3535
Coeff Var	347.75016		

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	4.19359	1.18771	3.53	0.0006
x1	1	1.20946	0.19079	6.34	<.0001
x2	1	-0.86335	0.20353	-4.24	<.0001

The REG Procedure
 Model: MODEL2
 Dependent Variable: outcome

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	9638.84645	9638.84645	61.02	<.0001
Error	110	17376	157.96243		
Corrected Total	111	27015			

Root MSE	12.56831	R-Square	0.3568
Dependent Mean	3.60714	Adj R-Sq	0.3510
Coeff Var	348.42842		

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	4.19946	1.19001	3.53	0.0006
diff	1	1.04678	0.13400	7.81	<.0001