ST 512, Summer Session II, Dr. Jason A. Osborne
Homework assignment #3

1. Recall exercise 2 from HW 2 (Rao 12.5) which looks at the data in the file "plantht1.dat". Use Tukey's procedure to conduct all 10 pairwise comparisons among means for the $t = 5$ treatment combinations of light type and intensity. Suppose you were only interested in these three contrasts:

$$\begin{aligned} \theta_1 &= \mu_2 + \mu_3 - (\mu_4 + \mu_5) \\ \theta_2 &= \mu_3 + \mu_5 - (\mu_2 + \mu_4) \\ \theta_3 &= \mu_3 - \mu_2 - (\mu_5 + \mu_4) \end{aligned}$$

   (a) Report estimates of each contrast, along with standard errors.

   (b) Report the sum of squares associated with each contrast.

   (c) Report simultaneous 95% confidence intervals for the three contrasts, using the Bonferroni correction.

2. Consider designing an experiment to evaluate the potential effectiveness of $t = 5$ weight reducing agents. Suppose that $n$ subjects are to be assigned at random to each of $t = 5$ treatment groups. Suppose that the smallest meaningful effect on weight loss that researchers involved in the study would like to detect is one in which the variance among the weight loss treatment means is at least as great as those given in the alternative below:

$$H_1 : \mu_A = \mu_E = 12, \mu_B = 11, \mu_C = 10, \mu_D = 9$$

   Suppose further that the standard deviation among weight losses for any treatment group is about $\sigma = 1$. Hold the type I error rate at $\alpha = 0.05$.

   (a) Compute the number of subjects necessary to obtain a power of at least $1 - \beta = 0.9$.

   (b) Obtain a plot of the power against sample sizes between 2 and 10.

   (c) Describe how the power would change if $\sigma$ were actually larger.

   (d) Describe how the power would change if the population mean weight gain for agent 1 were $\mu_1 = 15$.

   (e) Suppose the $n = 10$ is adopted, and data are observed as given in the table below.

Weight losses under five agents

| | A | B | C | D | E |
|---|---|---|---|---|---|
| mean | 12.05 | 11.02 | 10.27 | 9.27 | 12.17 |
| std.dev. | .83 | 1.12 | 1.03 | 1.16 | 0.79 |

      i. Carry out a test for the hypothesis that the treatment has no effect on weight loss. Use $\alpha = 0.05$

      ii. After carrying out all pairwise comparisons at familywise error rate $\alpha = .05$, identify which differences are significant. Be clear about which multiple comparison procedure you use.

      iii. Explain the difference between strong and weak control of the familywise error rate in this context.

      iv. Consider the complex contrast that compares the mean of agents $A$ and $E$ with that of agent $D$.

A. Express this (population) contrast as a vector product involving the vector of (population) treatment means, $\mu' = (\mu_A, \mu_B, \mu_C, \mu_D, \mu_E)$.

B. Report an estimate of the contrast from the data summarized in the table.

C. Report a standard error.

D. Report the sum of squares associated with the contrast.

3. (This problem is just for fun. No need to turn anything in.) Click the link on the ST511 website that takes you to the page of applets by Webster at Univ. of South Carolina. Then click the "Let's Make a Deal" link.

`www.stat.sc.edu/~west/applets/LetsMakeaDeal.html`.

Read the description of the problem. Now, consider an experiment where you get to observe $n$ independent $0-1$ trials and you are interested in competing hypotheses for the success probability $p$:

$$H_0 : p = 1/2 \quad \text{versus} \quad H_1 : p = 2/3$$

Let $Y$ denote the number of successes out of the $n$ trials.

(a) Suppose you observe $n = 30$ such trials and adopt this critical region for $Y$ :

$$\text{Reject } H_0 \text{ if } Y \geq 20.$$

Using Table C.5 or an appropriate BINOMIAL applet, obtain the exact significance level of this test.

(b) Using the applet by Lenth, find either the approximate or exact power of the test which uses the critical region we've specified.

`www.stat.uiowa.edu/~rlenth/Power/`.

(c) Collect your data using the switch strategy. Play the game $n = 30$ times and report your results. Are your data "statistically significant", in terms of the test you've set up? Briefly state your conclusion regarding $H_0$ and $H_1$, using $\alpha$ from part a).

4. (Rao 13.10 and Rao 13.18) A greenhouse study investigates the effects of two environmental conditions - amount of light (10 hr, 12 hr) and daily temperature ($70^o$ F) and ($80^o$ F) on growth of a certain species of plant. $n = 4$ plants were grown under each of the $2 \times 2 = 4$ conditions. Dryweights given online as "drywt.dat". (In this file, `x1` denotes amount of light (1 for 10 hrs, 2 for 12 hrs) and `x2` denotes temperature (1 for $70^o$, 2 for $80^o$).

(a) Perform a one-way ANOVA to test the null hypothesis that there is no difference between the effects of the four environmental condiitions.

(b) Estimate the effect of interaction between the amount of light and daily temperature.

(c) Test the significance of the contrast you estimated in (b).

(d) Calculate the sum of squares for interaction and for each main effect.

(e) Complete an ANOVA table for a $2 \times 2$ analysis of the data.

(f) Briefly summarize findings from the ANOVA table.

(g) Construct an interaction plot that may be used to help explain these conclusions.

5. (Rao 13.5 and Rao 13.23) In a greenhouse study, the drought resistance of four varieties of a field crop was compared. After a pretreatment to stimulate root growth, each cutting of a given variety was exposed to a drought condition. There were three pretreatments, and each pretreatment was applied to three cuttings. Data are available online as "root.dat". In the dataset, $i$ denotes pretreatment, $j$ cutting $x$ denotes variety and $y$ denotes root length.

   (a) Identify the factors and their levels.

   (b) Propose a statistical model for the root lengths, using factorial effects.

   (c) Carry out a one-way ANOVA of the data to see if there is a difference between expected root lengths for the $t = 12$ treatment combinations resulting from combining three types of pretreatments (levels of factor $A$) with four varieties (levels of factor $B$).

   (d) Construct an appropriate plot of the treatment means (an interaction plot). On the basis of the plot, comment on the existence of various types of factorial effects (such as interaction effects and main effects.)

   (e) Construct an ANOVA table needed to test the significance of the factorial effects.

   (f) Perform an appropriate factorial analysis of the data and write your conclusions.