ST 512, Summer Session II, Dr. Jason A. Osborne
Homework assignment #3

1. Recall exercise 2 from HW 2 (Rao 12.5) which looks at the data in the file "plantht1.dat". Use Tukey's procedure to conduct all 10 pairwise comparisons among means for the $t = 5$ treatment combinations of light type and intensity. Suppose you were only interested in these three contrasts:

$$\begin{aligned}\theta_1 &= \mu_2 + \mu_3 - (\mu_4 + \mu_5) \\ \theta_2 &= \mu_3 + \mu_5 - (\mu_2 + \mu_4) \\ \theta_3 &= \mu_3 - \mu_2 - (\mu_5 - \mu_4)\end{aligned}$$

(A) Using Tukey's HSD to conduct all 10 pairwise comparisons. Inspection of the output below indicates that any differences that contrast light types A and B are significant as is the difference $\bar{y}_D - \bar{y}_{AH}$, for a total of five significant differences, with strong control of $FWE = .05$.

```
The SAS System                                                        3
The GLM Procedure
Tukey's Studentized Range (HSD) Test for y

NOTE: This test controls the Type I experimentwise error rate, but it generally
has a higher Type II error rate than REGWQ.

(some output omitted)

Means with the same letter are not significantly different.

Tukey
Grouping
                Mean      N    trt
     A       34.3075      4    4
     A       34.2925      4    5
B    A       34.0200      4    1
B    C       31.9000      4    2
     C       30.8400      4    3
```

(a) Report estimates of each contrast, along with standard errors.

$$\begin{aligned}\hat{\theta}_1 &= -5.86(\widehat{SE} = 1.04) \\ \hat{\theta}_2 &= 1.075(\widehat{SE} = 1.04) \\ \hat{\theta}_3 &= -1.045(\widehat{SE} = 1.04)\end{aligned}$$

(b) Report the sum of squares associated with each contrast.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
theta1	1	34.33960000	34.33960000	31.45	<.0001
theta2	1	1.15562500	1.15562500	1.06	0.3199
theta3	1	1.09202500	1.09202500	1.00	0.3331

(c) Report simultaneous 95% confidence intervals for the three contrasts, using the Bonferroni correction.

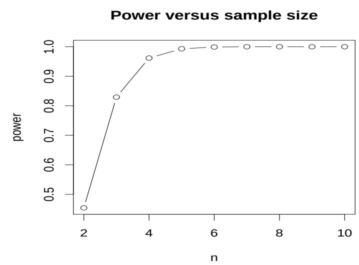
Parameter	98.33% Confidence Limits	
theta1	-8.67358731	-3.04641269
theta2	-1.73858731	3.88858731
theta3	-3.85858731	1.76858731

2. Consider designing an experiment to evaluate the potential ...

$$H_1 : \mu_A = \mu_E = 12, \mu_B = 11, \mu_C = 10, \mu_D = 9$$

(a) Compute the number of subjects necessary to obtain a power of at least $1 - \beta = 0.9$. For $n = 4$, a power of $1 - \beta = .962$ is achieved.

(b) Obtain a plot of the power against sample sizes between 2 and 10.



(c) Describe how the power would change if σ were actually larger. As σ increases, the power, $1 - \beta$ decreases.

(d) Describe how the power would change if the population mean weight gain for agent 1 were $\mu_1 = 15$. Other things being equal, this would increase the treatment effect, so that power would increase.

(e) Suppose the $n = 10$ is adopted, and data are observed as given in the table below.

Weight losses under five agents

	A	B	C	D	E
mean	12.05	11.02	10.27	9.27	12.17
std.dev.	.83	1.12	1.03	1.16	0.79

i. Carry out a test for the hypothesis that the treatment has no effect on weight loss. Use $\alpha = 0.05$.

Source	*df*	MS
Agent	4	14.97
Error	45	0.99

$F = 15.05, p < .0001$, the means differ significantly.

ii. After carrying out all pairwise comparisons at familywise error rate $\alpha = .05$, identify which differences are significant. Be clear about which multiple comparison procedure you use. I used Tukey to obtain an honestly significant difference of $HSD = q(.05, 5, 45)\sqrt{MSE/10} = 1.27$. In the table below, means with the same letter do not differ significantly:

Agent	Mean	Grouping
D	9.27	A
C	10.27	AB
B	11.02	BC
A	12.05	C
E	12.17	C

iii. Explain the difference between strong and weak control of the familywise error rate in this context. Strong control means $\Pr(\geq 1 \text{false positive}) < .05$ no matter what the population means are. Weak control only controls under the situation where all are equal. Consider the alternative studied in the power computations part of this problem:

$$H_1 : \mu_A = \mu_E = 12, \mu_B = 11, \mu_C = 10, \mu_D = 9.$$

Fisher's LSD will accidentally declare the difference between μ_A and μ_E significant more than desired, violating the FWE. Tukey's procedure will be ok for this comparison.

iv. Consider the complex contrast that compares the mean of agents A and E with that of agent D.

A. Express this (population) contrast as a vector product involving the vector of (population) treatment means, $\mu' = (\mu_A, \mu_B, \mu_C, \mu_D, \mu_E)$.

$$\theta = (\frac{1}{2}, 0, 0, -1, \frac{1}{2})\mu'$$

B. Report an estimate of the contrast from the data summarized in the table.

$$\hat{\theta} = \frac{1}{2}(\bar{y}_A + \bar{y}_E) - \bar{y}_D = \frac{1}{2}(12.05 + 12.17) - 9.27 = 2.84$$

C. Report a standard error.

$$SE(\hat{\theta}) = \sqrt{MSE(\frac{.5^2 + 0^2 + 0^2 + (-1)^2 + .5^2}{10})} = 0.39$$

D. Report the sum of squares associated with the contrast.

$$SS(\hat{\theta}) = \frac{\hat{\theta}^2}{(\frac{.5^2+0^2+0^2+(-1)^2+.5^2)}{10}} = 53.8$$

(Note that $SS[Trt] = 59.9$.)

3. (This problem is just for fun. No need to turn anything in.) Click the link on the ST511 website that takes you to the page of applets by Webster at Univ. of South Carolina. Then click the "Let's Make a Deal" link.

`www.stat.sc.edu/~west/applets/LetsMakeaDeal.html`.

Read the description of the problem. Now, consider an experiment where you get to observe n independent $0-1$ trials and you are interested in competing hypotheses for the success probability p:

$$H_0 : p = 1/2 \quad \text{versus} \quad H_1 : p = 2/3$$

Let Y denote the number of successes out of the n trials.

(a) Suppose you observe $n = 30$ such trials and adopt this critical region for Y :

$$\text{Reject } H_0 \text{ if } Y \geq 20.$$

Using Table C.5 or an appropriate BINOMIAL applet, obtain the exact significance level of this test.

(b) Using the applet by Lenth, find either the approximate or exact power of the test which uses the critical region we've specified.

`www.stat.uiowa.edu/~rlenth/Power/`.

(c) Collect your data using the switch strategy. Play the game $n = 30$ times and report your results. Are your data "statistically significant", in terms of the test you've set up? Briefly state your conclusion regarding H_0 and H_1, using α from part a).

4. (a) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$, $F = 1406.8, p < .0001, df = 3, 12$ (strong evidence of a treatment effect.)

(b) $\hat{\theta} = 14(SE = 0.59)$ (difference of slopes) also ok would be $\hat{\theta} = -14$ or $\hat{\theta} = 14/2$ or $\hat{\theta} = -14/2$, so long as the right SE is used. (The p-value for a test of $\theta = 0$ is the same for all of them.

Parameter	Estimate	Standard Error	t Value	Pr > |t|
interaction	-14.0500000	0.59196002	-23.73	<.0001

(c) $t = 23.7$ or $F = 563$ on $df = 1, 12$. Highly significant interaction.

(d) I used SAS to generate the sums of squares, though calculation by hand would not be too bad:

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
interaction	1	197.402500	197.402500	563.34	<.0001
ss(light)	1	222.010000	222.010000	633.56	<.0001
ss(temp)	1	1059.502500	1059.502500	3023.55	<.0001

(e) Standard

The GLM Procedure

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1478.915000	492.971667	1406.82	<.0001
Error	12	4.205000	0.350417		
Corrected Total	15	1483.120000			

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
interaction	1	197.402500	197.402500	563.34	<.0001
ss(light)	1	222.010000	222.010000	633.56	<.0001
ss(temp)	1	1059.502500	1059.502500	3023.55	<.0001

(f) From output below, there is evidence of interaction, so it would be wise to investigate either the simple effect of light duration separately for each temperature, or vice-versa.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1478.915000	492.971667	1406.82	<.0001
Error	12	4.205000	0.350417		
Corrected Total	15	1483.120000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
light	1	222.010000	222.010000	633.56	<.0001
temp	1	1059.502500	1059.502500	3023.55	<.0001
light*temp	1	197.402500	197.402500	563.34	<.0001

(g) 12 hours is more beneficial for growth than 10 hours, but only at the higher temperature. See plot below.

(h) Text plot (not pretty, but still gets the point across):

```
                    Plot of p*light.  Symbol is value of temp.

          40 +
             |
             |
             |
             |                                                2
             |
          30 +
             |
             |
           p |
             |
             |
          20 +  2
             |
             |
             |
             |
             |
          10 +  1                                             1
             ---+---------------------------------------------+--
                1                                             2

                                    light
```

5. (Rao 13.5 and 13.23)

(a)

Factor	# levels	levels
A: pretreatment	3	1,2,3
B: variety	4	1,2,3,4

(b) Let i denote pretreatment, j denote variety, k cutting.

$$\begin{aligned} Y_{ijk} &= \mu_{ij} + E_{ijk} \\ \mu_{ij} &= \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} \end{aligned}$$

E_{ijk} assumed iid $N(0, \sigma^2)$.

(c) $H_0 : \mu_{ij} \equiv \mu$ for all i, j. Observed $F = 8.52, p < .0001, df = 11, 24$. There is strong evidence of a treatment effect.

(d) Interaction plot below suggests variety effects, but not when pretreatment 3 is used. Generally, pretreatments 1 and 2 are better than pretreatment 3, though the advantage is less pronounced for variety 4.

```
          Plot of ymean*pretrt.  Symbol is value of variety.

   30 +
      |
      |
      |
      |3                              3
      |
   20 +                               2
      |2
      |
ymean |
      |
      |                               14
   10 +
      |1                                                             4
      |4
      |                                                              23
      |                                                              1
      |
    0 +
      -+------------------------------+------------------------------+
       1                              2                              3

                                   pretrt
```

(e) ANOVA table below

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	1872.555556	170.232323	8.52	<.0001
Error	24	479.333333	19.972222		
Corrected Total	35	2351.888889			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
pretrt	2	924.3888889	462.1944444	23.14	<.0001
variety	3	525.4444444	175.1481481	8.77	0.0004
variety*pretrt	6	422.7222222	70.4537037	3.53	0.0120

(f) The test for interaction is significant. I suppose one might want to test for simple pretreatment effects separately for each variety, since the plot suggests the effect might be muted for variety 4. Indeed a glance at the output below suggests as much.

Least Squares Means
variety*pretrt Effect Sliced by variety for y

variety	DF	Sum of Squares	Mean Square	F Value	Pr > F
1	2	112.666667	56.333333	2.82	0.0794
2	2	448.222222	224.111111	11.22	0.0004
3	2	724.666667	362.333333	18.14	<.0001
4	2	61.555556	30.777778	1.54	0.2346

Code used for this problem:

```
proc glm;
   class variety pretrt;
   model y=pretrt|variety;   /* two-factor analysis */
   lsmeans pretrt*variety/slice=variety;
   output out=two p=ymean;
run;

/*goptions device=pslepsf colors=black;*/

symbol1 i=join l=1 value=dot;
symbol2 i=join l=1 value=square;
symbol3 i=join l=1 value=diamond;
symbol4 i=join l=1 value=plus;

options ps=30 ls=70;

*proc plot;
proc gplot;
   title "Interaction plot for 13.23";
   plot ymean*pretrt=variety;
run;
```

6. iv injection system experiment

(a)

$$\hat{\theta} = \bar{y}_{1+} - \frac{1}{3}(\bar{y}_{2+} + \bar{y}_{3+} + \bar{y}_{4+}) = 35.5 - \frac{1}{3}(17.1 + 23.8 + 25.8) = 13.3$$

(b)

$$\widehat{SE}(\hat{\theta}) = \sqrt{MS(E)\left(\frac{1}{9} + \frac{(-1/3)^2 + (-1/3)^2 + (-1/3)^2}{9}\right)} = 0.95(not 1.2$$

95% c.i.

$$\hat{\theta} \pm t(.025, 24)\widehat{SE} \quad \text{or } 13.3 \pm 2.06(0.95)((not 1.2)) \quad \text{or} \quad 13.3 \pm 2.0$$