ST512

Fall Semester, 2007

Quiz 3

Name:_____

Directions:

- Answer questions as directed. Please show work, but not when the answer may be found directly in output. Partial credit may be awarded for correct expressions given in incomplete answers.
- Use the back of the page or the extra paper provided if necessary.
- By default, use level of significance $\alpha = 0.05$ on all testing or confidence interval problems, unless directed otherwise.
- If time is an issue, you do not need to complete elementary arithmetic, so that, for example, a confidence interval of the form $10.2 \pm 2.1\sqrt{\frac{2}{5}11.4}$ is an acceptible answer.
- Some problems here are based on experiments discussed in Yandell (1997) and others are taken from the R software package examples.

1. (20 pts) Eight corn-growing sites were randomly sampled from the island of Antigua. Harvest weights were measured on four equally sized parcels of land sampled from each of the eight sites, and are summarized below. Consider a one-factor random effects model to study variability among all corn-growing sites in Antigua.

	site i								row	row
	DB	LF	NS	OR	OV	TE	WE	WL	mean	variance
$\bar{y}_{i+}$	4.89	4.21	2.09	6.92	4.83	3.04	5.53	2.84	$\bar{y}_{++} = 4.29$	2.51
s_i^2	0.09	0.77	0.56	0.16	1.77	0.15	0.71	0.42	$8^{-1}\sum s_i^2 = 0.58$	

(a) Compute the site mean square, $MS(\text{site})$.

(b) Estimate the variance components for this model.

(c) Report an estimate of the coefficient of variation among parcels.

(d) Report a 95% confidence interval for the mean yield among all such parcels of this size on Antigua.

2. (30 pts) An experiment run by an herbal product company to study blood pressure (BP) reduction brought about by its product randomizes 10 subjects to treatment or placebo (5 each). Baseline BP is measured, the pill is used for a prescribed period and then measurements are collected. Since BP varies substantially from one day to the next, and even one hour to the next, measurements are made on ten consecutive visits after the treatment period, at random time-of-day. The response variable is reduction from baseline, y_{ijk}, visit k, subject j, treatment group i. SAS code and output for a (possibly inappropriate) model with fixed subject and treatment effects is given below.

```
proc glm;
   class trt subj ;
   model  sbp=trt subj(trt);
   means  trt;
run;
```

```
                              The SAS System                                1
                             The GLM Procedure

                Class         Levels    Values
                trt                2    placebo supplement
                subj               5    1 2 3 4 5

                                     Sum of
Source                     DF       Squares   Mean Square  F Value  Pr > F
Model                       9   2380.089560    264.454396     6.28  <.0001
Error                      90   3790.056375     42.111738
Corrected Total            99   6170.145936

Source                     DF   Type III SS   Mean Square  F Value  Pr > F
trt                         1    432.623013    432.623013    10.27  0.0019
subj(trt)                   8   1947.466547    243.433318     5.78  <.0001

           Level of              ---------sbp_decrease--------
           trt             N             Mean          Std Dev

           placebo        50       0.16503935       7.00494655
           supplement     50       4.32495768       8.24760743
```

(a) The p-value for `trt` is highly significant, indicating a treatment effect. Is it valid? Explain. If it is not valid, conduct the appropriate test as you see fit.

(b) For the model you believe to be appropriate for this experiment, estimate the treatment effect and any parameters that describe subject and visit variability.

(c) Give expressions for 95% confidence intervals for each parameter in part (b). (Use $\chi^2_{\alpha,\nu}$ to denote critical values from the χ^2 distribution.)

3. (20 pts) In a latin square design, five cows were each infused with five amounts of a digestion aid (0,0.5,1,1.5,2 kg/day) in (restrictedly) random order. Randomization to the five treatment amounts over five periods was such that exactly one of the cows received any given treatment level in any period, with a month between periods to avoid carryover effects. The mean responses (protein flow) are given below, along with the columnwise (single) sums of squared deviations about the grand mean $\bar{y}_{++} = 42$. The corrected total sum of squares for all $N = 25$ measurements is $\sum_i \sum_j (y_{ij} - \bar{y}_{++})^2 = 8200$.

column index $(i, j \text{ or } k)$	means by cow $\bar{y}_{i+}$	means by period $\bar{y}_{+j}$	means by infusion level $\bar{y}_k$
1	42	50	22
2	44	42	34
3	42	32	42
4	44	46	52
5	38	40	60
single sum of squared deviations	24	184	888

Consider an additive model with cow, period and infusion (treatment) effects.

(a) Report the F-ratio and df for a test that mean protein flow is constant across the five infusion levels

(b) Consider a model in which the effect of infusion level on protein flow is linear. (The linear contrast coefficients are $(-2, -1, 0, 1, 2)$.) Obtain the F-ratio and df for a test for lack-of-fit of this model.

4. (30 pts) Each of eight plots are subdivided into four smaller plots and sweet corn is planted in each at one of four dates, assigned at random. The plots themselves are randomly assigned to two insecticide treatments (control or treated) since the insecticide cannot be applied to units as small as the subplots. Root damage caused by Western Corn Rootworm is visually assessed for each of the $N = 32$ subplots. SAS output appears on the page entitled "ROOTWORM PROBLEM".

   (a) It is expected that both insecticide and plant date will affect the rootworm populations and hence the root damage they cause. Report F-ratios, along with associated *df* and p-values for tests for main effects of these factors.

   (b) Report a 95% confidence interval for the main effect of the insecticide treatment.

(c) Also of interest is whether the reduction in root damage due to insecticide diminishes with a later planting date.

i. For the hypothesis that reduction due to insecticide is constant across all four planting dates, report an F-ratio, df and p-value.

ii. Consider the contrast comparing this reduction at the first and last plant dates only. Obtain a 95% confidence interval for this contrast. Does the contrast differ significantly from 0?

(d) (6 points) Compile an ANOVA table with five columns: source of variability, df, sum of squares, mean square, and expected mean square. You may use the fact that $\sum_i \sum_j \sum_k (y_{ijk} - 2.39)^2 = 84.7$.

As a small holiday gift, you don't have to solve this problem (4d). Six points to all, and to all a happy holiday! (Also, good luck with your other exams.)

```
proc mixed order=data;
   class pdate treatment plot;
   model rootdamage=pdate|treatment/ddfm=satterth;
   random plot(treatment);
   lsmeans pdate*treatment;
run;
```


The Mixed Procedure

Class	Levels	Values
pdate	4	May24 June02 June13 June23
treatment	2	control insecticide
plot	8	1 2 3 4 5 6 7 8

Covariance Parameter Estimates

Cov Parm	Estimate
plot(treatment)	0.7632
Residual	0.8737

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
pdate	3	18	5.72	0.0063
treatment	1	6	6.21	0.0470
pdate*treatment	3	18	2.31	0.1104

Least Squares Means

Effect	pdate	treatment	Estimate	Standard Error	DF	t Value
treatment		control	3.2615	0.4954	6	6.58
treatment		insecticide	1.5153	0.4954	6	3.06
pdate*treatment	May24	control	4.6025	0.6397	14.5	7.19
pdate*treatment	May24	insecticide	1.9822	0.6397	14.5	3.10
pdate*treatment	June02	control	4.0171	0.6397	14.5	6.28
pdate*treatment	June02	insecticide	1.5584	0.6397	14.5	2.44
pdate*treatment	June13	control	2.6181	0.6397	14.5	4.09
pdate*treatment	June13	insecticide	1.1644	0.6397	14.5	1.82
pdate*treatment	June23	control	1.8085	0.6397	14.5	2.83
pdate*treatment	June23	insecticide	1.3564	0.6397	14.5	2.12

Table of critical values ($t(df, \alpha)$) from t-distributions:

df	$\alpha = 0.2$	$\alpha = 0.15$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$	$\alpha = 0.001$
1	1.37638	1.96261	3.07768	6.31375	12.7062	31.8205	63.6567	318.309
2	1.06066	1.38621	1.88562	2.91999	4.3027	6.9646	9.9248	22.327
3	0.97847	1.24978	1.63774	2.35336	3.1824	4.5407	5.8409	10.215
4	0.94096	1.18957	1.53321	2.13185	2.7764	3.7469	4.6041	7.173
5	0.91954	1.15577	1.47588	2.01505	2.5706	3.3649	4.0321	5.893
6	0.90570	1.13416	1.43976	1.94318	2.4469	3.1427	3.7074	5.208
7	0.89603	1.11916	1.41492	1.89458	2.3646	2.9980	3.4995	4.785
8	0.88889	1.10815	1.39682	1.85955	2.3060	2.8965	3.3554	4.501
9	0.88340	1.09972	1.38303	1.83311	2.2622	2.8214	3.2498	4.297
10	0.87906	1.09306	1.37218	1.81246	2.2281	2.7638	3.1693	4.144
11	0.87553	1.08767	1.36343	1.79588	2.2010	2.7181	3.1058	4.025
12	0.87261	1.08321	1.35622	1.78229	2.1788	2.6810	3.0545	3.930
13	0.87015	1.07947	1.35017	1.77093	2.1604	2.6503	3.0123	3.852
14	0.86805	1.07628	1.34503	1.76131	2.1448	2.6245	2.9768	3.787
15	0.86624	1.07353	1.34061	1.75305	2.1314	2.6025	2.9467	3.733
16	0.86467	1.07114	1.33676	1.74588	2.1199	2.5835	2.9208	3.686
17	0.86328	1.06903	1.33338	1.73961	2.1098	2.5669	2.8982	3.646
18	0.86205	1.06717	1.33039	1.73406	2.1009	2.5524	2.8784	3.610
19	0.86095	1.06551	1.32773	1.72913	2.0930	2.5395	2.8609	3.579
20	0.85996	1.06402	1.32534	1.72472	2.0860	2.5280	2.8453	3.552
21	0.85907	1.06267	1.32319	1.72074	2.0796	2.5176	2.8314	3.527
22	0.85827	1.06145	1.32124	1.71714	2.0739	2.5083	2.8188	3.505
23	0.85753	1.06034	1.31946	1.71387	2.0687	2.4999	2.8073	3.485
24	0.85686	1.05932	1.31784	1.71088	2.0639	2.4922	2.7969	3.467
25	0.85624	1.05838	1.31635	1.70814	2.0595	2.4851	2.7874	3.450
26	0.85567	1.05752	1.31497	1.70562	2.0555	2.4786	2.7787	3.435
27	0.85514	1.05673	1.31370	1.70329	2.0518	2.4727	2.7707	3.421
28	0.85465	1.05599	1.31253	1.70113	2.0484	2.4671	2.7633	3.408
29	0.85419	1.05530	1.31143	1.69913	2.0452	2.4620	2.7564	3.396
30	0.85377	1.05466	1.31042	1.69726	2.0423	2.4573	2.7500	3.385
40	0.85070	1.05005	1.30308	1.68385	2.0211	2.4233	2.7045	3.307
60	0.84765	1.04547	1.29582	1.67065	2.0003	2.3901	2.6603	3.232
80	0.84614	1.04320	1.29222	1.66412	1.9901	2.3739	2.6387	3.195
100	0.84523	1.04184	1.29007	1.66023	1.9840	2.3642	2.6259	3.174
200	0.84342	1.03913	1.28580	1.65251	1.9719	2.3451	2.6006	3.131
100000	0.84162	1.03644	1.28156	1.64487	1.9600	2.3264	2.5759	3.090

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