ST512

Fall Semester, 2007

Quiz 3

Name:_____

Directions:

- Answer questions as directed. Please show work, but not when the answer may be found directly in output. Partial credit may be awarded for correct expressions given in incomplete answers.
- Use the back of the page or the extra paper provided if necessary.
- By default, use level of significance $\alpha = 0.05$ on all testing or confidence interval problems, unless directed otherwise.
- If time is an issue, you do not need to complete elementary arithmetic, so that, for example, a confidence interval of the form $10.2 \pm 2.1\sqrt{\frac{2}{5}11.4}$ is an acceptible answer.
- Some problems here are based on experiments discussed in Yandell (1997) and others are taken from the R software package examples.

1. (20 pts) Eight corn-growing sites were randomly sampled from the island of Antigua. Harvest weights were measured on four equally sized parcels of land sampled from each of the eight sites, and are summarized below. Consider a one-factor random effects model to study variability among all corn-growing sites in Antigua.

	site i								row	row
	DB	LF	NS	OR	OV	TE	WE	WL	mean	variance
$\bar{y}_{i+}$	4.89	4.21	2.09	6.92	4.83	3.04	5.53	2.84	$\bar{y}_{++} = 4.29$	2.51
s_i^2	0.09	0.77	0.56	0.16	1.77	0.15	0.71	0.42	$8^{-1}\sum s_i^2 = 0.58$	

(a) Estimate the variance components for this model.

$$\hat{\sigma}^2 = MS(E) = 0.58$$

$$\hat{\sigma}_S^2 = \frac{MS(\text{site}) - MS(E)}{4}$$

$$MS(\text{site}) = \frac{1}{7}\sum\sum(\bar{y}_{i+} - \bar{y}_{++})^2 = 10.04$$

$$\hat{\sigma}_S^2 = \frac{10.04 - 0.58}{4} = 2.365$$

(b) Report an estimate of the intrasite correlation (two parcels from the same site).
$2.365/(2.365 + 0.58) = 0.8$

(c) Report an estimate of the coefficient of variation among parcels.

$$\frac{\hat{\sigma}_s^2 + \hat{\sigma}^2}{\hat{\mu}} = \frac{\sqrt{2.365 + 0.58}}{4.29} = 0.4 = 40\%$$

(d) Report a 95% confidence interval for the mean yield among all such parcels of this size on Antigua.

$$\bar{y}_{++} \pm t(.025, 7)\sqrt{\frac{MS(\text{site})}{32}}$$

or

$$4.29 \pm 2.36(0.56) \quad \text{or} \quad 4.29 \pm 1.32$$

2. (30 pts) In a trial . . .

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	2380.089560	264.454396	6.28	<.0001
Error	90	3790.056375	42.111738		
Corrected Total	99	6170.145936			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
trt	1	432.623013	432.623013	10.27	0.0019
subj(trt)	8	1947.466547	243.433318	5.78	<.0001

Level of trt	N	sbp_decrease Mean	sbp_decrease Std Dev
control	50	0.16503935	7.00494655
supplement	50	4.32495768	8.24760743

(a) The p-value for `trt` is highly significant, indicating a treatment effect. Is it valid? Explain. If it is not valid, conduct the appropriate test as you see fit. The p-value above assumes all 100 observations are independent. A more reasonable model would include random subject effects, nested in treatment, leading to

$$F_{trt} = \frac{MS(trt)}{MS(subj(trt))} = \frac{432.6}{243.4} = 1.78$$

which is not at all significant.

(b) For the model you believe . . . estimate the treatment effect and any parameters required to describe subject and visit variability.

$$\begin{aligned} \hat{\theta} &= \bar{y}_{2++} - \bar{y}_{1++} \\ &= 4.32 - 0.17 = 4.15 \\ \sigma^2 &= MS(E) = 42.1(\text{ visit-to-visit variability}) \\ \sigma_{subj} &= \frac{MS(subj(trt)) - MS(E)}{10} \\ &= \frac{243.4 - 42.1}{10} \approx 20 \end{aligned}$$

(c) Give expressions for 95% confidence intervals for each parameter in part (b).

$$\hat{\theta} \pm t(.025, 8)\sqrt{\frac{2}{50}MS(subj(trt))}$$

$$\left(\frac{90MS(E)}{\chi^2_{.975,90}}, \frac{90MS(E)}{\chi^2_{.025,90}}\right) \quad \text{or} \quad (32, 57.7)$$

$$\widehat{df} = \frac{(\sum c_i MS_i)^2}{\sum (c_i MS_i)^2/df_i} = 5.4$$

$$\left(\frac{\widehat{df}\hat{\sigma}^2_s}{\chi^2_{.975,\widehat{df}}}, \frac{\widehat{df}\hat{\sigma}^2_s}{\chi^2_{.025,\widehat{df}}}\right)$$

3. (20 pts) In a latin square design, ...

column index $(i, j \text{ or } k)$	means by cow $\bar{y}_{i+}$	means by period $\bar{y}_{+j}$	means by infusion level $\bar{y}_k$
1	42	50	22
2	44	42	34
3	42	32	42
4	44	46	52
5	38	40	60
single sum of squared deviations	24	184	888

Consider an additive model with treatment, cow and period effects.

(a) Report the F-ratio for a test that the mean protein flow is constant across the five infusion levels

$$SS(E) = SS[Tot] - SS[cow] - SS[period] - SS[trt] = 8200 - 5(24) - 5(184) - 5(888) = 2720$$

$$MS(E) = SS(E)/df_E = 2720/12 = 227$$

$$F = \frac{MS(trt)}{MS(E)} = \frac{4440/4}{227} = \frac{1110}{227} = 4.89$$

(b) Consider a model in which the effect of infusion level on protein flow is linear. Test for lack-of-fit of this model.

$$\hat{\theta}_{linear} = -2\bar{y}_1 - \bar{y}_2 + 0\bar{y}_3 + \bar{y}_4 + 2\bar{y}_5 = 94$$

$$SS(\hat{\theta}_{linear}) = \frac{94^2}{\frac{10}{5}} = 4418$$

$$F_{LOF} = \frac{(SS(trt) - SS(\hat{\theta}_{linear}))/(4-1)}{MS(E)} = \frac{(4440 - 4418)/3}{227} = .03$$

(no lack-of-fit).

4. (30 pts) Each of eight plots are subdivided into four smaller plots and sweet corn is planted in each at one of four dates, assigned at random. The plots themselves are randomly assigned to two insecticide treatments (control or treated) since the insecticide cannot be applied to units as small as the subplots. Root damage caused by Western Corn Rootworm is visually assessed for each of the $N = 32$ subplots. SAS output appears on the page entitled "ROOTWORM PROBLEM".

   (a) It is expected that both insecticide and plant date will affect the rootworm populations and hence the root damage they cause. Report F-ratios, along with associated df and p-values for tests for main effects of these factors. From output,

$$F_{trt} = 6.21 \quad df = 1, 6, \quad p = .0470$$
$$F_{pdate} = 5.72 \quad df = 3, 18 \quad p = .0063$$

   (b) Report a 95% confidence interval for the main effect of the insecticide treatment.

$$\bar{y}_{1++} - \bar{y}_{2++} \pm t(.025, 6)\sqrt{\frac{2}{16}MS(plot(trt))}$$

   or

$$1.75 \pm 2.45\sqrt{3.93/8} \quad \text{or} \quad 1.75 \pm 1.72$$

   (c) Also of interest is whether the reduction in root damage due to insecticide diminishes with a later planting date.

      i. Test the hypothesis that reduction due to insecticide is constant across all four planting dates. Report an F-ratio, df and p-value. From output, $F = 2.31, df = 3, 18, p = .1104$

      ii. Consider the contrast comparing this reduction at the first and last plant dates only. Obtain a 95% confidence interval for this contrast. Does the contrast differ significantly from 0?

$$\hat{\theta} = \bar{y}_{14+} - \bar{y}_{24+} - (\bar{y}_{11+} - \bar{y}_{21+} = 2.17$$
$$SE(\hat{\theta}) = \sqrt{\frac{4}{4}MS(E)} = .93$$

   since $t(.025, 18) = 2.1$, we get

$$2.17 \pm 2.1(.93) \quad \text{or} \quad 2.17 \pm 1.96$$

   (d)

source	df	sum of squares	ms	ems
treatment	1	24.4	24.4	$16\psi^2_{trt} + 4\sigma^2_s + \sigma^2$
plot(treatment)	6	23.6	3.9	$4\sigma^2_s + \sigma^2$
pdate	3	15	5	$8\psi^2_{pdate} + \sigma^2$
treatment × pdate	3	6.1	2.0	$4\psi^2_{pdate\times trt} + \sigma^2$
error	18	15.7	0.8737	σ^2
total	35			

```
proc mixed order=data;
   class pdate treatment plot;
   model rootdamage=pdate|treatment/ddfm=satterth;
   random plot(treatment);
   lsmeans pdate*treatment;
run;
```


The Mixed Procedure

Class	Levels	Values
pdate	4	May24 June02 June13 June23
treatment	2	control insecticide
plot	8	1 2 3 4 5 6 7 8

Covariance Parameter Estimates

Cov Parm	Estimate
plot(treatment)	0.7632
Residual	0.8737

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
pdate	3	18	5.72	0.0063
treatment	1	6	6.21	0.0470
pdate*treatment	3	18	2.31	0.1104

Least Squares Means

Effect	pdate	treatment	Estimate	Standard Error	DF	t Value
treatment		control	3.2615	0.4954	6	6.58
treatment		insecticide	1.5153	0.4954	6	3.06
pdate*treatment	May24	control	4.6025	0.6397	14.5	7.19
pdate*treatment	May24	insecticide	1.9822	0.6397	14.5	3.10
pdate*treatment	June02	control	4.0171	0.6397	14.5	6.28
pdate*treatment	June02	insecticide	1.5584	0.6397	14.5	2.44
pdate*treatment	June13	control	2.6181	0.6397	14.5	4.09
pdate*treatment	June13	insecticide	1.1644	0.6397	14.5	1.82
pdate*treatment	June23	control	1.8085	0.6397	14.5	2.83
pdate*treatment	June23	insecticide	1.3564	0.6397	14.5	2.12