ST512

Fall Quarter, 2005

Exam 3

Name:_____

Directions: Answer questions as directed. Show work where appropriate. For true/false questions, circle either true or false.

1. (40 pts) 25 men are randomly sampled from a population of interest and diastolic blood pressures is measured on each of 16 occasions. Let y_{ij} denote the measurement for subject i and occasion j. Summary statistics are given below.

$$\bar{y}_{++} = 91.7, \quad \sum_{i=1}^{25}\sum_{j=1}^{16}(\bar{y}_{i+} - 91.7)^2 = 11529, \quad \sum_i \sum_j (\bar{y}_{ij} - \bar{y}_{i+})^2 = 19541$$

Consider a model with subject effects A_i assumed i.i.d. $N(0, \sigma_1^2)$ and errors E_{ij} assumed i.i.d. $N(0, \sigma_2^2)$, independent of $\{A_i\}$.

(a)

$$\begin{aligned} df_E &= 25(16-1) = 375 \\ MSE &= 19541/375 \\ &= 52 \\ &= \hat{\sigma}_2^2 \\ df_A &= 25 - 1 = 24 \\ MS[A] &= 11529/24 \\ &= 480.4 \\ \hat{\sigma}_1^2 &= \frac{MS[A] - MSE}{16} \\ &= \frac{480.4 - 52}{16} \\ &= 26.8 \end{aligned}$$

(b) Use the normal approximation to the chisquare distribution ($\chi^2 n \approx N(n, 2n)$) to report an approximate 95% confidence interval for the variance component quantifying day-to-day variability (or variability not due to subject). The approximation $SSE/\sigma_2^2 \dot{\sim} N(375, 750)$ leads to the 95% confidence interval

$$\left(\frac{SSE}{428.6}, \frac{SSE}{321.3}\right) \quad \text{or} \quad (45.6, 60.8)$$

(c) Estimate the intraclass correlation coefficient and give two interpretations of this parameter estimate.

$$\hat{\rho}_I = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_1^2 + \hat{\sigma}_2^2} = \frac{26.8}{26.8 + 52.1}$$

(d) Estimate the mean diastolic blood pressure among males. Report a 95% confidence interval for this parameter. A 95% confidence interval given by

$$\bar{y}_{++} \pm t(.025, 24)\sqrt{MS[A]/400} \text{ or } 91.7 \pm 2.26$$

2. (30 pts) An experiment investigates the effect of row spacing on soybean yields. The design uses five row spacings (18,24,30,36 or 42 inches apart) in a randomized complete block design with 6 plots. SAS code and output appropriate for this experiment appear at the end of this exam under the title "SOYBEAN PROBLEM".

   (a) Report the F-ratio and associated degrees of freedom for a test that row spacing has no effect on yield.

$$F = \frac{MS[spacing]}{MSE} = \frac{126.5/4}{63.9/20} = 9.9 \text{ on } \; df = 4, 20$$

   (b) Report Tukey's minimum significant difference (to control experimentwise error at $\alpha = .05$) for all pairwise comparisons among row spacing means.

$$q(.05, 5, 20)\sqrt{\frac{MSE}{6}} = 4.23\sqrt{3.2/6} = 3.09$$

   (c) The coefficients for the linear polynomial contrast are $(-2, -1, 0, 1, 2)$. Obtain the F-ratio and associated degrees of freedom for a test of lack of fit in a model in which yield is linearly related to row spacing.

$$\begin{aligned}
SS(\theta) &= \frac{\hat{\theta}^2}{10/6} \\
&= 96.8 \\
SS(LOF) &= (SS(\text{spacing} - SS(\theta)) \\
&= (126.5 - 96.8) \\
&= 30.2 \\
F_{LOF} &= \frac{SS(LOF/(4-1))}{MSE} \\
&= 10.1/3.2 \\
&= 3.14 \text{ on } df = 3, 20
\end{aligned}$$

3. (30 pts) An experiment investigates the effects of two factors on corn yield. Three levels of nitrogen (56, 112, and 168 lbs/ha) and two types of irrigation (furrow, sprinkler) are used in a balanced 3×2 randomized design with a total of $N = 24$ plots. SAS code and output appropriate for this problem appears on a page entitled "CORNYIELDS PROBLEM" at the end of the exam.

   (a) Report the F-ratio and p-value for a test that any effects of nitrogen on yield are constant across irrigation types.

$$F_{irr*nit} = 8.32, p = .0028$$

   (b) For each level of nitrogen, estimate the simple effect of irrigation type. Report standard errors and p-values for a test of no simple effects in each case.

Nitrogen	irr effect	SE	p
56	4	4.14	0.35
112	26	4.14	$< .0001$
168	7	4.14	0.11

   (c) For sprinkler irrigation, is the effect of nitrogen nonlinear? Report the quadratic (nonlinear) polynomial contrast for nitrogen (with coefficients $(1, -2, 1)$) when sprinkler irrigation is used. Report the standard error.

$$\begin{aligned} \hat{\theta}_{quad,sprinkler} &= \bar{y}_{1+} - 2\bar{y}_{2+} + \bar{y}_{3+} \\ &= = -39 \\ SE &= \sqrt{6MSE/4} \\ &= 7.2 \end{aligned}$$

(d) Does the nonlinearity of the response to nitrogen vary across irrigation types? The quadratic polynomial contrast for furrow-irrigated corn is $\hat{\theta}_{quad,furrow} = 2$. Report the difference between the quadratic contrasts for furrow and sprinkler irrigated corn. Report the standard error for this difference.

$$\begin{aligned} \hat{\theta}_{quad,sprinkler} - \hat{\theta}_{quad,furrow} &= 2 - (-39) \\ &= 41 \\ SE &= \sqrt{2MSE * 6/4} \\ &= 10.1 \end{aligned}$$

(e) Draw a conclusion about the effects of nitrogen and irrigation type on yield, chacterizing the nature of their interaction. Sketch a plot if it helps. The effect is linear for furrow-irrigated, but nonlinear for sprinkler irrigated. In particular, it looks like the sprinkler offers a big improvement with 112 lbs/ha of nitrogen.

```
proc glm;
class block spacing;
model y=spacing block;
means spacing/tukey;
```


The GLM Procedure

Class Level Information

Class	Levels	Values
block	6	1 2 3 4 5 6
spacing	5	18 24 30 36 42

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	?	133.4333333			
Error	??	71.5333333			
Corrected Total	29	204.9666667			

R-Square	Coeff Var	Root MSE	y Mean
0.651000	6.029355	1.891208	31.36667

Source	DF	Type I SS	Mean Square	F Value	Pr > F
spacing	?	126.4666667			
block	?	6.9666667			

Tukey's Studentized Range (HSD) Test for y

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	??
Error Mean Square	??
Critical Value of Studentized Range	4.23186
Minimum Significant Difference	?????

Means with the same letter are not significantly different.

Tukey Grouping	Mean	N	spacing
A	35.167	6	18
B	31.833	6	24
B	30.333	6	30
B	30.000	6	42
B	29.500	6	36

```
proc glm order=dat;
   class irrig nit;
   model yield=irrig|nit;
   lsmeans irrig*nit/slice=nit;
   estimate "furrow: quad nit   " nit 1 -2 1 irrig*nit 1 -2 1;
```


The GLM Procedure

Class	Levels	Values
irrig	2	furrow sprinkler
nit	3	56 112 168

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	4539.333333	907.866667	26.53	<.0001
Error	18	616.000000	34.222222		
Corrected Total	23	5155.333333			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
irrig	1	912.666667	912.666667	26.67	<.0001
nit	2	3057.333333	1528.666667	44.67	<.0001
irrig*nit	2	569.333333	284.666667	8.32	0.0028

Least Squares Means

irrig	nit	yield LSMEAN
furrow	56	160.000000
furrow	112	171.000000
furrow	168	184.000000
sprinkler	56	164.000000
sprinkler	112	197.000000
sprinkler	168	191.000000

irrig*nit Effect Sliced by nit for yield

nit	DF	Sum of Squares	Mean Square	F Value	Pr > F
56	1	32.000000	32.000000	0.94	0.3464
112	1	1352.000000	1352.000000	39.51	<.0001
168	1	98.000000	98.000000	2.86	0.1078

Parameter	Estimate	Standard Error	t Value	Pr > |t|
furrow: quad nit	2.00000000	7.16472842	0.28	0.7833