

**ST512****Fall Quarter, 2004****Exam III, due Monday, November 22****You may pick four of the five problems to solve****Each will be worth 25 points**

1. An agronomist wants to compare yields of furrow-irrigated, sprinkler-irrigated and non-irrigated soybeans. He has  $N = 9$  plots to work with, but knows from past experience that even for fixed irrigation treatment, variability in yield is high due to the planting history of the plot and the variety of soybeans that are raised. So, he puts the plots into three groups of three according to their rank in last years' planting density: low, medium or high. Within each of these groups, he distributes the three varieties (Porgate, Akona, and Clark62). He sets up the varieties in the plots in such a way that all  $3 \times 3 = 9$  combinations of prior planting density and variety are used. He randomizes the three irrigation treatments to the  $N = 9$  plots in such a way that each irrigation each treatment is tested on exactly on of these  $3 \times 3 = 9$  combinations of prior planting density and variety. The data, including yields, treatments, prior densities and varieties, along with code to read them into a SAS dataset are available as "soybeans-exam3.sas".
  - (a) Name the experimental design used here. (After inspection of the data, you may want to draw a diagram to clarify the design.)
  - (b) Obtain an ANOVA table appropriate for testing for the effect of irrigation treatment on yield.
  - (c) Test for an irrigation effect.
  - (d) Report the mean yield for the three irrigation treatments.
  - (e) Derive and report the standard errors of these means under the assumption that variety and prior planting density effects are fixed.

2. A small horticultural study with  $N = 25$  pecan trees investigates the effect of fertilizer application date on yield. Before application, the experimenter is able to put trees into 5 groups based on their performance last year; (very poor, poor, average, good, very good). The five treatments in the experiment are control or date of a single application of fertilizer: Control, July 1, Aug 1, Sept 1 or Oct 1. Within each of the 5 groups, 5 trees are randomized to the 5 treatments. The response,  $y$ , is yield in *lbs.* the following year.

Data and relevant SAS code can be found at the ST512 website under the name `pecans-exam3.sas`. (*The data, originally collected by Bill Goff, Auburn University, have been modified for this exam.*)

- (a) Propose a statistical model to analyze the data from this experiment. Treat the prior performance factor as a fixed effect. Use terminology developed in the course to name the design of the experiment, for purposes of reference.
- (b) Report an appropriate ANOVA table.
- (c) Report the estimated mean yields for each treatment. Report an estimated standard error for each.
- (d) Sketch a plot of the mean response on the vertical axis against the treatments, meaningfully ordered along the horizontal axis.
- (e) Formulate and test relevant hypotheses from the ANOVA table, using level of significance  $\alpha = 0.05$ .
- (f) Consider the following family of comparisons: all pairwise comparisons among fertilized (non-control) treatment means and a comparison of the control against the average of the four fertilized treatments.
  - i. Estimate all seven of these contrasts and report standard errors.
  - ii. Among the Tukey, Scheffe and Bonferroni multiple comparison techniques, choose one that controls the familywise error rate at  $\alpha = 0.05$ . Among those that control the fwe, choose the one that results in confidence intervals that are the most narrow.
  - iii. Use the procedure you've chosen to indicate which of the  $k = 7$  contrasts are significant at  $fwe = 0.05$ .
- (g) The researcher, a stickler for the  $\alpha = 0.05$  level of significance, sees that the  $p$ -value for a test of `group` effects is not significant. He wants to drop `group` from the model, reasoning that he'll increase the degrees of freedom for error and thereby increase the power of the test. What do you say?

3. A researcher wants to conduct an experiment to measure how well three compounds stimulate appetites in sheep. He believes that food intakes for normal, juvenile sheep observed over an 8-hour period are normally distributed about a mean of  $\mu = 200g$ . When asked to give a range which contains the middle 50% of the population of food intakes he replies about 180 up to 220 so that  $\sigma \approx 30$ . He has three injection treatments to which he'd like to randomize  $N = 15$  sheep: saline, orexin-B and CRF. Consider such an experiment carried out to test for a treatment effect on food intake, using level  $\alpha = 0.05$ . You may assume all  $N = 15$  measurements are independent and normally distributed.

(a) Consider the  $H_1$  hypothesis that the treatment group means are

$$H_1 : \mu_s = 200, \mu_{OB} = 250 \text{ and } \mu_{CRF} = 240.$$

- i. Compute the power of the  $F$ -test to detect such a treatment effect on food intake.
- ii. Compute the type II error rate for such a test under  $H_1$ .
- iii. Suppose that the researcher has underestimated the variability of food intakes and that the middle 50% of the population really ranges from 160 up to 240, for an interquartile range of 80 and a standard deviation of  $\sigma \approx 60$ . The power of the experiment to detect  $H_1$  would then be (bigger/smaller) than that computed in part (i).
- iv. Suppose the researcher had  $N = 21$  sheep to randomize to treatments. The power of the experiment to detect  $H_1$  would then be (bigger/smaller) than that computed in part (i).
- v. If the researcher is unwilling to accept a significance level of  $\alpha = 0.05$  and will only declare the differences among treatment means to be significant if the  $p$ -value for the  $F$ -ratio in part (i) is less than 0.01. The power of the experiment to detect  $H_1$  would then be (bigger/smaller) than that computed in part (i).
- vi. Obtain a plot of the power on the vertical axis against the treatment sample size  $n$  on the horizontal axis.

4. A food science student conducts an experiment to study the (fixed) effects of type of oven and amount of energy on the “offnote” taste of peanuts. She uses two oventypes (conventional/microwave) and three power levels ( $1x/2x/3x$ ) which are equally spaced. She produces  $n = 2$  batches of peanuts using each of the  $3 \times 2 = 6$  combinations of oventype and power and then has a panel of trained experts taste them, one-per-day, over  $N = 12$  days. She randomly assigns the batches to days. The response is average taste reported by the panel, on a scale from 1 to 15, with 15 indicating a strong “offnote” or bad taste. The data are available online as “offnote.dat”.
- (a) Report the  $F$ -ratio and  $p$ -value for a test that all 6 treatments have the same mean offnote flavor.
  - (b) Using appropriate greek symbols, specify the model for the mean offnote flavor using the factorial effects parameterization.
  - (c) Report the  $F$ -ratio and  $p$ -value for a test of a interaction between oventype and power level. Interpret the result as it pertains to peanuts.
  - (d) Consider the linear polynomial contrast,  $\theta_{LM}$ , for the effect of power level on microwaved peanuts.
    - Express this contrast in terms of parameters used in part (b)
    - Estimate this contrast.
    - Report a standard error.
    - Obtain the sum of squares for the contrast.
    - Report a  $p$ -value for a test comparing a model in which taste of microwaved peanuts is linear in power with a model in which it is constant.
  - (e) Consider the linear polynomial contrast,  $\theta_{LC}$ , for the effect of power level on peanuts cooked in a conventional oven.
    - Express this contrast in terms of parameters used in part (b)
    - Estimate this contrast.
    - Report a standard error.
    - Obtain the sum of squares for the contrast.
    - Report a  $p$ -value for a test comparing a model in which taste of microwaved peanuts is linear in power with a model in which it is constant.
  - (f) Consider the contrast  $\theta_{LM} - \theta_{LC}$ .
    - Estimate this contrast.
    - Report a standard error.
    - Report a  $p$ -value for a test that the linear effect of power on taste is the same for both oven types.
  - (g) Obtain and interpret an interaction plot for the experiment.

5. It is suggested to the Ohio Department of Motor Vehicles that some people tolerate alcohol better than others. They conduct an experiment where they recruit volunteers, solicit their informed consent, then observe their performance in hand-eye coordination tests after drinking alcoholic beverages. In particular, they'll take the test, then drink a beer, then take the test again fifteen minutes after taking the beer, then drink another beer and then take the test in another fifteen minutes. The response is the cumulative drop in coordination test scores, and is measured in two separate sessions, one week apart. You may assume that the drops are normally distributed and that the subjects are a random sample of adults.

Session	Subject								sample
	DJF	VF	AC	AT	BQ	JM	HT	LN	mean
Week 1	-3	0	2	3	4	2	4	8	$\bar{y}_1 = 2.5$
Week 2	-5	0	4	1	3	1	5	7	$\bar{y}_2 = 2.0$

- (a) Consider a random effects model for this experiment. Conduct an  $F$ -test to investigate the claim that some adults tolerate alcohol better than others. Report an appropriate  $p$ -value and indicate the degrees of freedom associated with the test.
- (b) Consider the variance component which reflects variability in alcohol tolerance due to subject.
- Report an unbiased method-of-moments estimate of this variance component.
  - Obtain a confidence interval for the variance component based on this estimator.
- (c) Consider the variance component which reflects variability in alcohol tolerance within subject.
- Report an unbiased method-of-moments estimate of this variance component.
  - Obtain a confidence interval for the variance component based on this estimator.
- (d) Report the estimated intrasubject correlation of drop-in-coordination-test measurements.
- (e) Consider the mean drop in test scores after two beers and 30 minutes.
- Report an unbiased estimate,  $\hat{\mu}$ , of this mean drop in test score.
  - Use  $\hat{\mu}$  to estimate the coefficient of variation for a randomly sampled subject and session.
  - Report a 95% confidence interval for the mean drop in test score.