ST512 - Osborne

An old exam problem on two-factor fixed effect ANOVA

Run the SAS code in the file "bp.sas" and use the output to answer the questions below:

1. A study was performed to look at systolic blood pressure (BP) in three different diet groups:

   - strict vegetarians (SV)
   - lactovegetarians (LV) (eat dairy products)
   - normal - (standard American diet)

   Samples of size $n = 5$ from each gender were taken from each dietary group and BP was measured. Total sample size is $N = 30$. Let y_{ijk} denote the BP for subject k in diet group i and gender j.

   (a) Estimate the standard deviation of BPs among people in a given dietary-gender classification group, assuming homogeneity of variance across the 6 groups.

   (b) True/false: The $MS[E]$ is the average of the sample variances for the 6 treatment combinations:

   $$MS[E] = \frac{1}{6}(s_{11}^2 + s_{12}^2 + s_{21}^2 + s_{22}^2 + s_{31}^2 + s_{32}^2)$$

   where $s_{ij}^2 = \frac{1}{4}\sum_{k=1}^{5}(y_{ijk} - \bar{y}_{ij+})^2$.

   (c) Test for a diet×gender interaction effect at level $\alpha = 0.05$.

   (d) Report a p-value for the null hypothesis that the average BPs in the three diet populations are equal.

(e) Estimate the difference between mean BP for men and women. Report a standard error.

(f) True/False: There is significant ($\alpha = 0.05$) evidence to suggest that population differences among diet groups are different for men than for women.

(g) Let $\mu_{NOR}, \mu_{LV}, \mu_{SV}$ denote the mean BPs in the three populations after averaging over gender. Prepare a table of estimates, with estimated standard errors in parentheses:

$$\begin{aligned} \hat{\mu}_{NOR} &= \qquad\qquad (\qquad\qquad) \\ \hat{\mu}_{SV} &= \qquad\qquad (\qquad\qquad) \\ \hat{\mu}_{LV} &= \qquad\qquad (\qquad\qquad) \end{aligned}$$

(h) Suppose you are interested in $c = 4$ contrasts: the 3 pairwise contrasts and the difference θ between mean BP for normal eaters and the average of the mean BPs for the two vegetarian populations. Use Scheffé's procedure to obtain simultaneous 95% confidence intervals for these four contrasts. Note that

$$\boxed{\begin{aligned} \sqrt{(3-1)F(0.95,2,24)MS[E]\left(\frac{2}{10}\right)} &= 10.1 \\ \sqrt{(3-1)F(0.95,2,24)MS[E]} &= 22.5 \\ \sqrt{\frac{1}{10}+\frac{1/4}{10}+\frac{1/4}{10}} &= 0.4 \end{aligned}}$$

$$\begin{aligned} \mu_{NOR} - \mu_{SV}: &\quad 11.0 \pm 10.1 \\ \mu_{NOR} - \mu_{LV}: &\quad \pm \\ \mu_{LV} - \mu_{SV}: &\quad \pm \\ \theta: &\quad \pm \end{aligned}$$

(i) Label each sum of squares below so that the source of variability which it quantifies is clear

$$SS[diet], SS[gender], SS[diet \times gender], SS[E], SS[Tot]$$

Sum of squares	Label
$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{5} (y_{ijk} - \bar{y}_{ij+})^2$	
$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{5} (\bar{y}_{i++} - \bar{y}_{+++})^2$	
$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{5} (\bar{y}_{ij+} - \bar{y}_{i++} - \bar{y}_{+j+} + \bar{y}_{+++})^2$	
$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{5} (\bar{y}_{+j+} - \bar{y}_{+++})^2$	
$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{5} (y_{ijk} - \bar{y}_{+++})^2$	

(j) Complete the interaction plot below, using "M" for male, "F" for female.