ST512
Summer Session II, 2008
Quiz 3-solutions

1. An industrial engineer investigates the effect of three different types of computer mouse on repetitive motion disorders (RMD). She is able to recruit $N = 12$ subjects, sampled from a population of interest, and randomly assign them to the three mouse types for a period of two weeks. Each subject records the amount of time they spend using the mouse (z, in hours) and their subjective assessment of RMD pain, y.

   (a) Ignoring hours of usage z altogether, conduct an F-test for a treatment (mouse type) effect on RMD pain. Use level $\alpha = .05$. You may use the fact that $F(.05, 2, 9) = 4.26$. (You will need to recompute $MS(E)$.)

   $$F = \frac{MS(Trt)}{MS(E)} = \frac{2558/3}{(5149 - 2558)/9} = \frac{1279}{288} = 4.44$$

   This F-ratio is barely significant, indicating some evidence of an effect of mouse type on pain.

   (b) Again, test for a mouse type effect on pain, but this time after controlling for hours of usage in an appropriate analysis of covariance model. Report the p-value. Directly from the output, $F = 4.96, p = .0397, df = 2, 8$, still providing some evidence of a mouse type effect, after controlling for hours of usage.

   (c) Report a table of adjusted and unadjusted means.

Mouse type	Sample Mean	Adjusted Mean
Reflective	73	68.9
Rolltrack	37.5	47
ronventional	59	52.7

   (d) Provide an estimate of the mean difference in RMD pain between the conventional mouse and the roll-track mouse, under both the ANCOVA model of part (b) and the naive one-way ANOVA of part (a) Under ANCOVA model, $\hat{\beta}_2 = -5.7$. Under ANOVA model, $\hat{y}_{Ro} - \bar{y}_c = 37.5 - 59 = -21.5$.

   (e) Report standard errors for the difference in part (d) under both models. From output, $\widehat{SE}(\hat{\beta}_2) = 7.75$, for ANOVA estimate, $\widehat{SE}(\hat{y}_{Ro} - \bar{y}_c) = \sqrt{MS(E)\frac{2}{4}} = \sqrt{288/2} = 12$.

2. In an experiment to study the competitive effect of weed growth on grass, $N = 24$ plots are randomly assigned to $t = 6$ treatments, with $n = 4$ plots per treatment. A fescue of interest is allowed to establish itself in all plots, and then a measured amount of a particular weed is introduced after one year. After another year, the fraction of living material in each plot that is fescue is measured, with the results below:

	Treatment					
	N_1	Y_1	N_2	Y_2	N_3	Y_3
Mean	94	83	81	68.5	53	49
sd	3	3.5	3.5	5.3	3.8	4.4
	A	B	B	C	D	D

(a) After pooling data across treatments, estimate the variance of grass fraction for a fixed treatment. $MS(E) = \frac{1}{6}(3^2 + 3.5^2 + 3.5^2 + 5.3^2 + 3.8^2 + 4.4^2) = 15.9$

(b) Use Tukey's procedure to identify those sample treatment means which differ significantly at familywise error rate $\alpha = .05$. (You may annotate the table above.) $HSD = q(.05, 6, 18)\sqrt{MS(E)/4} = 4.49\sqrt{15.9/4} = 8.96$ Pairwise differences larger than 8.96 are significant at familywise error rate $\alpha = .05$. (See table above.)

(c) Explain what familywise error rate means here. Be very brief. The probability of at least one type I error (aka "false positive") among the family of all $\binom{6}{2} = 15$ pairwise comparisons.

(d) Suppose that not all 15 possible pairwise contrasts are important. Rather, a smaller number $k < 15$ of comparisons are of interest. If a data analyst is trying to choose between the Bonferroni and Tukey procedures, for what values of k is the Bonferroni procedure preferred? Above, we saw that $HSD = 8.96$. For $k = 9$, using the Bonferroni correction we'd have $\alpha' = \alpha/9$ and minimum significant difference (MSD) $MSD = t(.025/9, 18)\sqrt{MS(E)(2/4)} = 8.88$ but as k goes up to 10, $MSD = t(.025/10, 18)\sqrt{MS(E)(2/4)} = 9.01$, so that Bonferroni is preferred so long as $k \leq 9$. The thing to compare is

$$\frac{q(.05, 6, 18)}{\sqrt{2}} \lessgtr t(.025/k, 18)$$

because the other parts of the HSD and MSD cancel.

(e) Estimate a contrast comparing grass fraction for the average of treatments 1 and 2 with that for the average of treatments 5 and 6. Report a standard error.

$$\begin{aligned}
\hat{\theta} &= \frac{1}{2}(\bar{y}_{1+} + \bar{y}_{2+}) - (\bar{y}_5 + \bar{y}_{6+})) \\
&= 37.5 \\
\widehat{SE} &= \sqrt{MS(E)(\frac{(1/2)^2 + (1/2)^2 + (1/2)^2 + (1/2)^2}{4})} \\
&= 1.99
\end{aligned}$$

3. Consider a completely randomized experiment with $t = 4$ treatments, and n experimental units randomized to each treatment, for a total of $4n$ observations. For the questions below, assume the factorial effects model $Y_{ij} = \mu + \tau_i + E_{ij}$ where E_{ij} are i.i.d. normal with mean 0, unknown variance σ^2. The null hypothesis is $H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$. For parts (a) (b) and (c), circle the correct answer.

   (a) For fixed error variance σ^2 and a given alternative hypothesis, H_1, what happens to the *power* of the experiment to reject H_0 in favor of H_1 as the sample size n becomes larger?

   - it goes up
   - it goes down
   - it stay the same

   (b) For fixed σ and n, what happens to the type II error rate, β as one considers treatment means that are further and further apart (larger and larger $|\tau_i|$)?

   - it goes up
   - it goes down
   - it stay the same

   (c) Suppose the smallest alternative you'd like to be able to find is one where $\mu_1 = 65, \mu_2 = 58, \mu_3 = 64, \mu_4 = 49$ so that $\tau_1 = 6, \tau_2 = -1, \tau_3 = 5, \tau_4 = 10$. Assume a population standard deviation of $\sigma = 12$. With $n = 5$ experimental units per treatment, what is the sampling distribution of the statistic $F = MS(Trt)/MS(E)$ under the smallest alternative described above? (Circle one.)

   - Central F with 4 and 5 numerator and denominator degrees of freedom
   - Central F with 3 and 16 numerator and denominator degrees of freedom
   - Non Central F with 4 and 3 numerator and denominator degrees of freedom
   - Non Central F with 3 and 16 numerator and denominator degrees of freedom

   (d) Use software or the applet to find the power associated with these settings (H_1 above, $\sigma = 12$, $n = 5$). Does your experiment have a 50-50 or greater chance of detecting differences this big or bigger? (Briefly explain any use of software, including what values were input.) Using Lenth's applet, we get a standard deviation of the treatment effects that is

$$\text{SD[Trt]} = \sqrt{\frac{1}{4-1}\sum \tau^2} = \sqrt{\frac{1}{4-1}(6^2 + (-1)^2 + 5^2 + (-10)^2)} = 7.35$$

   and we set `SD[Within]` equal to 12, and $n = 5$ and get a power of $1 - \beta = 0.39$. The experiment has less than 50% chance to find the effect.