Statistical consulting - Osborne - Sept, 2003
Sanjay Shah's experiment on water quality
and turkey litter application treatments

The layout of the experiment can be viewed as a split-plot design with three treatments randomized to nine whole-plots. Within each plot, three rainfall events are applied, but without randomization to split-plots (as far as I know.) A statistical model for a response variable Y_{ijk}, (e.g. nutrient concentration) in such a design is given by

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + S_{k(i)} + E_{ijk}$$

where α_i denote fixed litter application treatment effects, β_j denote fixed rainfall event effects, $(\alpha\beta)_{ij}$ their interaction effects and $S_{k(i)}$ denote random plot effects, which are assumed i.i.d. $N(0, \sigma_s^2$ and E_{ijk} denote random error terms, assumed i.i.d. $N(0, \sigma^2)$. There was one missing value for a total of $3 \times 3 \times 3 - 1 = 26$ observations. The ANOVA table for this analysis, along p-values and error terms used in the appropriate F-ratios, are given below. (Non-integer coefficients are due to the missing value.)

Source	Error Term	Error DF	F Value	Pr > F
time	MS(Residual)	11	51.77	<.0001
trt	0.9832 MS(plot(trt)) + 0.0168 MS(Residual)	6.1386	20.41	0.0019
time*trt	MS(Residual)	11	13.74	0.0003
plot(trt)	MS(Residual)	11	1.48	0.2697
Residual	.	.	.	.

Conclusions from split-plot analysis:

1. Since the treatment $\times$ event interaction is highly significant ($p = 0.0003$ on approximately 6 and 11 degrees of freedom) treatment effects are investigated separately for each event:

   - In event 1 (natural rainfall of $8mm$ 2 days after treatment), there was no evidence of a treatment effect ($p = 0.3312$).
   - In event 2 (simulated rainfall of $47mm$ a few hours after natural rainfall), the treatment effect was highly significant $p < 0.0001$, and all three pairwise differences were significant. The estimated means are given below:

Effect	trt	time	Estimate	Standard Error	DF
time*trt	1	2	1.9000	0.2384	11
time*trt	2	2	4.0361	0.2901	11
time*trt	3	2	0.5400	0.2384	11

- In event 3 (simulated rainfall of $67mm$ 7 days after treatment), there was no evidence of a treatment effect ($p = 0.5448$).

2. The effect of the rainfall event was highly significant. When investigated separately for each treatment, the rainfall event was significant for treatments 1 and 2, where the differences were due to large mean responses after event 2. The rainfall effect was not significant for treatment 3.

Tests of Effect Slices

Effect	trt	time	Num DF	Den DF	F Value	Pr > F
time*trt	1		2	11	15.99	0.0006
time*trt	2		2	11	54.68	<.0001
time*trt	3		2	11	0.37	0.7006

3. These conclusions can be clarified by inspection of the estimated mean response at each treatment × event ("time") combination:

Effect	trt	time	Estimate	Standard Error	DF	t Value	Pr > |t|
time*trt	1	1	0.3467	0.2384	11	1.45	0.1738
time*trt	2	1	0.8000	0.2384	11	3.36	0.0064
time*trt	3	1	0.3400	0.2384	11	1.43	0.1815
time*trt	1	2	1.9000	0.2384	11	7.97	<.0001
time*trt	2	2	4.0361	0.2901	11	13.91	<.0001
time*trt	3	2	0.5400	0.2384	11	2.27	0.0447
time*trt	1	3	0.4033	0.2384	11	1.69	0.1187
time*trt	2	3	0.6600	0.2384	11	2.77	0.0183
time*trt	3	3	0.2867	0.2384	11	1.20	0.2544

The estimated variance component for plot was small $\sigma_s^2 = 0.0249$ compared to the error variance, $\sigma^2 = 0.1456$. and observed plot-to-plot variability was not statistically significant $p = 0.2697$.

SAS coding

If event is contained in a variable called `time`, then the output used in these analyses can be generated using SAS with the following code:

```
proc mixed method=type3;
   class time trt plot ;
   model y=time trt time*trt;
   random plot(trt);
   lsmeans trt*time/pdiff adj=tukey slice=time;
   lsmeans trt*time/pdiff adj=tukey slice=trt;
run;
```