

Analysis of Peacock and Rutledge experiments
on growth under cold conditions

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The dry weights of a total of $N = 96$ plugs were measured. The experimental factors under investigation are summarized below:

- A: Cultivar ($a = 8$ levels)
- B: Temperature ($b = 2$ levels)
- C: Replication ($c = 6$ levels for each cultivar)

Differences of dry weight from plugs grown in a green house (GH) are used as responses in the analysis:

$$D_i = Y_{GH} - Y_i \text{ for } i = 1, 2$$

Here i denotes the level of temperature -1° or -4° . These differences are therefore correlated through their common dependence on the control resulting in a split-plot design (with replications as plots) with cultivar as a between-rep factor and temperature as a within-rep factor. The full model can then be stated

$$D_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + S_{k(i)} + E_{j(ik)}$$

where α_i denote CV effects, β_i denote temperature effects, $(\alpha\beta)_{ij}$ denote CV -by-temperature interaction effects, $S_{k(i)}$ denote random replication effects (nested in CV), and $E_{j(ik)}$ denote residual errors. The replication and residual error terms in this mixed model are assumed to be independent and normally distributed with variance components σ_s^2 and σ^2 , respectively. These variance components are denoted in the SAS output by `Var(Rep(CV))` and `Var(Residual)`, respectively.

The analysis of variance summarized in Table 1 below indicates significant cultivar and temperature effects, with temperature effect not differing significantly by cultivar. Note that the error terms used in the denominators for the F -ratios are not the same for all fixed effects of interest (though the degrees of freedom associated with each is 40.)

Type 3 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error		
						DF	F Value	Pr > F
cv	7	0.650371	0.092910	Var(Residual) + 2 Var(Rep(cv)) + Q(cv,cv*temp)	MS(Rep(cv))	40	4.35	0.0012
temp	1	0.031393	0.031393	Var(Residual) + Q(temp,cv*temp)	MS(Residual)	40	9.16	0.0043
cv*temp	7	0.050094	0.007156	Var(Residual) + Q(cv*temp)	MS(Residual)	40	2.09	0.0672
Rep(cv)	40	0.854858	0.021371	Var(Residual) + 2 Var(Rep(cv))	MS(Residual)	40	6.24	<.0001
Residual	40	0.137023	0.003426	Var(Residual)

The estimated variance components shown below indicate that most of the error variance is coming from replications, and that the intra-rep correlation for two observations constructed by subtracting the same control dry weight is about $0.0090/0.0124 \approx 0.73$:

Covariance Parameter Estimates

Cov Parm	Estimate
Rep(cv)	0.008973
Residual	0.003426

Marginal means for the eight cultivars and two temperatures appear in Table 2 below. Two cultivar means with the same letter in the rightmost column do not differ significantly, according to multiple comparisons carried out using Tukey's procedure. In particular, there are three significant differences: *Celebration* and *TifSport*, *Celebration* and *Navy Blue*, and *Tifton 1* and *TifSport*. The standard error for any difference of sample cultivar means is

$$SE(\bar{Y}_{i_1..} - \bar{Y}_{i_2..}) = \sqrt{\frac{MS(Rep(CV))}{12}} = 0.060.$$

The estimate of the effect of a change of 3° temperature on mean dryweight reduction is $0.252 - 0.216 = 0.036$ with a standard error of $\sqrt{2MSE/48} = 0.012$ on 40 *df*. Note that Tukey's procedure controls the experimentwise error rate at $\alpha = 0.05$.

Effect	cv	temp	Estimate	Standard Error	DF
cv	Celebrat		0.3683	0.04220	40 A
cv	Tifton 1		0.3538	0.04220	40 A B
cv	Patriot		0.2292	0.04220	40 A B C
cv	Tifway		0.2215	0.04220	40 A B C
cv	GN-1		0.2189	0.04220	40 A B C
cv	Quicksta		0.2083	0.04220	40 A B C
cv	Navy Blu		0.1688	0.04220	40 B C
cv	TifSport		0.1057	0.04220	40 C
temp		-4	0.2524	0.01607	52.5
temp		-1	0.2162	0.01607	52.5

Some treatments at the most extreme temperature resulted in complete mortality, and thus 0 variability in dry weight measurements. The analysis based on differences from greenhouse measurements overcomes this difficulty, however, since all of the resulting differences exhibit variability.

Diagnostic plots of residuals indicate that the assumption of normally distributed data is reasonable. The residual (r) versus fitted (p) plot below indicates some inhomogeneity of variance, with variability of the effects apparently decreasing with mean effect. No remedial transformations, such as ratios or log ratios are possible because of the complete mortality treatments.

