Analysis of Peacock and Rutledge experiments
on growth under cold conditions
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The dry weights of a total of $N = 96$ plugs were measured. The experimental factors under investigation are summarized below:

A: Cultivar ($a = 8$ levels)
B: Temperature ($b = 2$ levels)
C: Replication ($c = 6$ levels for each cultivar)

Differences of dry weight from plugs grown in a green house (GH) are used as responses in the analysis:

$$D_i = Y_{GH} - Y_i \text{ for } i = 1, 2$$

Here i denotes the level of temperature -1^o or -4^o. These differences are therefore correlated through their common dependence on the control resulting in a split-plot design (with replications as plots) with cultivar as a between-rep factor and temperature as a within-rep factor. The full model can then be stated

$$D_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + S_{k(i)} + E_{j(ik)}$$

where α_i denote CV effects, β_i denote temperature effects, $(\alpha\beta)_{ij}$ denote CV-by-temperature interaction effects, $S_{k(i)}$ denote random replication effects (nested in CV), and $E_{j(ik)}$ denote residual errors. The replication and residual error terms in this mixed model are assumed to be independent and normally distributed with variance components σ_s^2 and σ^2, respectively. These variance components are denoted in the SAS output by `Var(Rep(CV))` and `Var(Residual)`, respectively.

The analysis of variance summarized in Table 1 below indicates significant cultivar and temperature effects, with temperature effect not differing significantly by cultivar. Note that the error terms used in the denominators for the F-ratios are not the same for all fixed effects of interest (though the degrees of freedom associated with each is 40.)

Type 3 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	Expected Mean Square	Error Term	Error DF	F Value	Pr > F
cv	7	0.497533	0.071076	Var(Residual) + 2 Var(Rep(cv)) + Q(cv,cv*temp)	MS(Rep(cv))	40	3.68	0.0037
temp	1	0.010004	0.010004	Var(Residual) + Q(temp,cv*temp)	MS(Residual)	40	22.08	<.0001
cv*temp	7	0.013270	0.001896	Var(Residual) + Q(cv*temp)	MS(Residual)	40	4.18	0.0015
Rep(cv)	40	0.772805	0.019320	Var(Residual) + 2 Var(Rep(cv))	MS(Residual)	40	42.64	<.0001
Residual	40	0.018124	0.000453	Var(Residual)	.	.	.	.

The estimated variance components shown below indicate that most of the error variance is coming from replications, and that the intra-rep correlation for two observations constructed by subtracting the same control dry weight is quite high at $0.0090/0.0099 \approx 0.95$:

Covariance Parameter Estimates

Cov Parm	Estimate
Rep(cv)	0.009434
Residual	0.000453

Marginal means for the eight cultivars and two temperatures appear in Table 2 below. Two cultivar means with the same letter in the rightmost column do not differ significantly, according to multiple comparisons carried out using Tukey's procedure. In particular, there are four significant differences involving *Celebration*, *Tifton 1*, *TifSport* and *Navy Blue*. The standard error for any difference of sample cultivar means is

$$SE(\overline{Y}_{i_1\cdot\cdot} - \overline{Y}_{i_2\cdot\cdot}) = \sqrt{\frac{MS(Rep(CV))}{12}} = 0.05675.$$

The estimate of the effect of a change of 3^o temperature on mean dryweight reduction is $0.2524 - 0.2320 = 0.0204$ with a standard error of $\sqrt{2MSE/48} = 0.004345$ on 40 *df* so that the temperature effect is highly significant, though not as pronounced as some of the culivar effects. Note that Tukey's procedure controls the experimentwise error rate at $\alpha = 0.05$.

Least Squares Means

Effect	cv	temp	Estimate	Standard Error	DF	
cv	Celebrat		0.3683	0.04012	40	A
cv	Tifton 1		0.3538	0.04012	40	A
cv	Patriot		0.2292	0.04012	40	A
cv	Tifway		0.2215	0.04012	40	A
cv	GN-1		0.2189	0.04012	40	A
cv	Quicksta		0.2083	0.04012	40	A
cv	Navy Blu		0.1688	0.04012	40	B
cv	TifSport		0.1687	0.04012	40	B
temp		-4	0.2524	0.01435	41.9	
temp		-1	0.2320	0.01435	41.9	

Some treatments at the most extreme temperature resulted in complete mortality, and thus 0 variability in dry weight measurements. The analysis based on differences from greenhouse measurements overcomes this difficulty, however, since all of the resulting differences exhibit variability.

Diagnostic plots of residuals indicate that the assumption of normally distributed data is reasonable. The residual (r) versus fitted (p) plot below indicates slight inhomogeneity of variance, with variability of the effects apparently decreasing with mean effect. No remedial transformations, such as ratios or log ratios are possible because of the complete mortality treatments.