Pared-down writeup of statistical analysis
Luck and Foegeding data on caramel color
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Data are collected in two single-factor experiments. In one experiment (#1), three color responses (*lh ah bh*) are measured using four different corn syrups (26,42,55,62) and a fixed level of phospholipid (30). In the other (#2) experiment, the color measurements are taken at six different levels of a phospholipid (0,1,5,10,20,30) using corn syrup #42 .

Data are analyzed using an unbalanced one-way fixed effects model with nine treatments. A total of 60 observations are taken for each color response, with 6 replications per treatment, with the exception of the treatment common to both experiments with phospholipid 30 and corn syrup #42, where 12 replicates are observed. An additional effect, nested in the treatments, is included to account for a possible experiment effect. The statistical model

$$Y_{ijk} = \mu + (\alpha\beta)_{ij} + \beta z_{ij} + E_{ijk}$$

where i indexes phospholipid, j indexes corn syrup and z_{ij} denotes an experiment #2 indicator variable is fit using the GLM procedure of SAS (1998).

Parametric hypotheses used in testing for phospholipid, corn syrup and experiment effects, are summarized in Table 1.

Table 1: Parametric tests of effects of interest

Effect	Hypothesis of no effect
Corn syrup	$H_0 : (\alpha\beta)_{61} = (\alpha\beta)_{62} = (\alpha\beta)_{63} = (\alpha\beta)_{64} = 0$
Phospholipid	$H_0' : (\alpha\beta)_{12} = (\alpha\beta)_{22} = \cdots = (\alpha\beta)_{62} = 0$
Experiment	$H_0'' : \beta = 0$

Appropriate F-ratios, computed using the SLICE option of an LSMEANS statement within PROC GLM, and corresponding numerator degrees of freedom and $p-$values for these hypotheses are summarized in Table 2.

Table 2: Results for tests of effects

Response	F and p for H_0	*df*	F and p for H_0'	*df*	F and p for H_0''
lh	73.24 ($p < .0001$)	3	2736.97 ($p < .0001$)	5	2.23 ($p = 0.1413$)
ah	102.86 ($p < .0001$)	3	2652.79 ($p < .0001$)	5	240.97 ($p < .0001$)
bh	18.76 ($p < .0001$)	3	31.56 ($p < .0001$)	5	3.90 ($p = 0.0538$)

Table 2 indicates that for each response variable, the simple effect of corn syrup when phosholipid is 30 ($i = 6$) is highly significant, as is the simple effect of phospholipid when cornsyrup 42 ($j = 2$) is used. There is strong evidence ($p < 0.0001$) of an experiment effect for the *ah* response: the means for the treatment with phospholipid #30 and corn syrup #42 are 8.39 for experiment #1 and 6.74 for #2.

The treatment means for the three responses are given in table 3 below:

Table 3: Estimated treatment means*

Treatment Number	Level of plipid	Level of cornsyrup	Number of reps.	lh Mean	ah Mean	bh Mean
1	0	42	6	69.19 a	-1.16 a	11.18 a
2	1	42	6	46.38 cd	0.75 b	13.05 bc
3	5	42	6	46.18 cd	3.88 c	13.36 bc
4	10	42	6	44.76 d	5.45 d	12.65 bc
5	20	42	6	45.65 cd	7.41 g	12.32 ab
6	30	26	6	52.09 b	6.18 e	17.38 e
7	30	42	12	47.22 c	7.57 g	15.54 d
8	30	55	6	42.05 e	7.62 g	14.03 c
9	30	62	6	46.75 cd	6.66 f	15.81 d

*means in the same column with the same letter are not significantly different

Subsequent multiple comparisons among the corn syrup means are made using a Bonferroni adjustment for multiplicity. These comparisons are made using the root mean squared error term from the ANOVA of the three responses lh, ah and bh which are $(1.2, 0.2, 0.8)$ respectively. Table 3 indicates that for corn syrup \$42, the mean responses with phospholipid 0 are significantly different from those for every other level of phospholipid. Similarly, for phospholipid fixed at 30, the sample mean of each response for cornsyrup #26 is significantly different those of the other three corn syrups.

Lastly, normal plots and residual-by-predicted plots of residuals for the color measurements did not reveal any obvious asymmetry or inhomogeneity of variance.

Appendix: SAS code

```
data one;
   input plipid  cornsyrup  rep  lh  ah  bh;
   if _n_ <= 36 then expt=1;
   else expt=2;
   cards;
0  42  1  66.17  -1.11  12.18
0  42  2  70.96  -1.21  10.95
0  42  3  70.55  -1.19  10.56
0  42  4  66.17  -1.1  12.19
0  42  5  70.91  -1.2  11.03
0  42  6  70.37  -1.15  10.17
1  42  1  44.84  0.93  12.56
.
.
.
30  55  6  42.65  7.55  14.4
;
run;

proc sort;
   by descending expt;
run;
proc glm order=data;
   title "nested";
   class plipid cornsyrup expt;
   model lh ah bh=plipid*cornsyrup expt(plipid*cornsyrup)/solution;
   lsmeans plipid*cornsyrup/slice=plipid pdiff adj=bon;
   lsmeans plipid*cornsyrup/slice=cornsyrup;
   lsmeans expt(plipid*cornsyrup);
   means expt(plipid*cornsyrup) plipid*cornsyrup;
run;
```