Interval estimation of nest survival rate

Consultant: Jason A. Osborne

Client: Andre Podolsky

DSR is the daily survival rate. It is estimated by the number of surviving (S) birds out divided by birds at risk (N). It is also $1 - F/N$ where F is the number of birds that perish:

$$\widehat{DSR} = \frac{S}{N} = 1 - \frac{F}{N}.$$

S is modelled as a binomial count so that the standard error of DSR is best estimated by

$$\widehat{SE}_1 = \widehat{SE}(D\hat{S}R) = \sqrt{\frac{\frac{S}{N}(1 - \frac{S}{N})}{N}}.$$

For large n (S and F each at least 5), $\widehat{DSR}$ is approximately normally distributed so that an approximate 95% confidence interval for DSR is given by

$$\widehat{DSR} \pm 2\widehat{SE}(\widehat{DSR})$$

or

$$\frac{S}{N} \pm 2\sqrt{\frac{\frac{S}{N}(1 - \frac{S}{N})}{N}}$$

(here the 97.5^{th} percentile of the standard normal distribution is about 2.

The nest survival rate for a period of k days is defined as

$$NSR(k) = DSR^k.$$

For large k, the distribution of this quantity will have a long left tail in the $(0, 1)$ interval. The log transform of $NSR(k)$ will be more normally distributed:

$$\log(\widehat{NSR}(k)) = k \log\left(\frac{S}{N}\right).$$

The approximate standard error of $\log(\widehat{NSR}(k))$ can be estimated using the delta method:

$$\widehat{SE}_2 = \widehat{SE}(\log(\widehat{NSR}(k)) = \frac{kN}{S}\sqrt{\frac{\frac{S}{N}(1 - \frac{S}{N})}{N}}.$$

An approximate 95% confidence interval for $\log(NSR(k))$ is just

$$k \log\left(\frac{S}{N}\right) \pm 2\frac{kN}{S}\sqrt{\frac{\frac{S}{N}(1 - \frac{S}{N})}{N}}.$$

(continued, next page)

This can safely be exponentiated to get an approximate 95% confidence interval for $NSR(k)$:

$$\left((\frac{S}{N})^k * e^{-2\widehat{SE}_2} < NSR(k) < (\frac{S}{N})^k * e^{2\widehat{SE}_2} \right).$$

For the case where $N = 1000, S = 950, F = 50, k = 30$ this yields the interval $(0.139, 0.332)$. It may seem wide, but it needs to be this wide. This procedure is almost optimally efficient.