

Power investigation for John Lowery's denitrification experiment

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For starters, the following fixed effects model was fit

$$Y_{ijkl} = \mu + \alpha_i + \gamma_j + \alpha\gamma_{ij} + \chi_{k(ij)} + E_{ijkl}$$

where Y denotes the sum of the measurements after 1 hour and after 2 hours and $i, j, k = 1, 2$ denote levels of fertilizer, location and chamber respectively:

$$i = \begin{cases} 1 & \text{unfertilized} \\ 2 & \text{fertilized} \end{cases}$$

$$j = \begin{cases} 1 & \text{fairway} \\ 2 & \text{rough} \end{cases}$$

$$l = \begin{cases} 1 & 1^{st} \text{ chamber} \\ 2 & 2^{nd} \text{ chamber} \end{cases}$$

The effect of chamber k is regarded as being randomly sample from a population of chamber effects and so modeled using a random effect. The analysis, based on $n_T = 16$ sums from the pilot data is summarized below:

The SAS System
The GLM Procedure
Class Level Information

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Class	Levels	Values
chamber	4	1 2 3 4
fert	2	1 2
location	2	1 2

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	1644234267	234890610	20.06	0.0002
Error	8	93670363	11708795		
Corrected Total	15	1737904630			

R-Square	Coeff Var	Root MSE	y Mean
0.946102	20.00175	3421.812	17107.56

Source	DF	Type III SS	Mean Square	F Value	Pr > F
fert	1	1450448183	1450448183	123.88	<.0001
location	1	136089723	136089723	11.62	0.0092
fert*location	1	51685316	51685316	4.41	0.0688
chambe(fert*locatio)	4	6011046	1502762	0.13	0.9678

Level of fert	Level of location	N	Mean	Std Dev
1	1	4	8705.5000	2526.53933
1	2	4	6467.2500	3729.80030
2	1	4	31342.5000	3255.27649
2	2	4	21915.0000	1528.23428

Covariance Parameter Estimates

Cov Parm	Estimate
chambe(fert*locatio)	0
Residual	8306784

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
fert	1	4	174.61	0.0002
location	1	4	16.38	0.0155
fert*location	1	4	6.22	0.0672

For these data, there was a significant fertilizer effect ($p = 0.0002$) and a significant location effect ($p = 0.0155$). The fertilizer effect was more pronounced at location 1 (fairway), but the interaction was not quite significant ($p = 0.0672$). Inference would be slightly different if the random effect of chamber had been ignored. Such an analysis is not unreasonable in light of the minimal observed chamber effect. There is no evidence of intrachamber correlation, so that an assumption of independence is reasonable. In a fixed-effects two-way factorial analysis with interaction, the p values for fertilizer, location and their interaction become $p = 0.0001, 0.0016, 0.0282$, respectively.

To determine appropriate sample sizes for future data collection, power was computed via simulation using a model with the following cell means

	Location	
	Fair	Rough
Unfertilized	8700	6500
Fertilized	30000	22000

The experimental errors used in the power simulation was taken from the estimates from the analyses with and without a chamber effect: $\sigma = 3400, 2900$ respectively. For each fertilizer \times location treatment combination, $n_{ij} \equiv 3$ was used. The tables on the following page describe the results. They were generated using the %SimPower Macro for SAS developed in Westfall, et al (1999).

Method=TUKEY, Nominal FWE=0.05, nrep=400, Seed=1234 1
 True means = (8700,6500,30000,22000), n=3, s=3400

Quantity	Estimate	---95% CI----
Complete Power	0.0225	(0.008,0.037)
Minimal Power	1.0000	(1.000,1.000)
Proportional Power	0.7250	(0.713,0.737)
Directional FWE	0.0025	(0.000,0.007)

Method=TUKEY, Nominal FWE=0.05, nrep=400, Seed=1234 1
 True means = (8700,6500,30000,22000), n=3, s=2900

Quantity	Estimate	---95% CI----
Complete Power	0.0325	(0.015,0.050)
Minimal Power	1.0000	(1.000,1.000)
Proportional Power	0.7650	(0.755,0.775)
Directional FWE	0.0025	(0.000,0.007)

The term *minimal power* indicates the probability of rejecting the complete null hypothesis that all four treatment means are equal.

References:

Westfall, Peter H., Tobias, Randall D., Rom, Dror, Wolfinger, Russell D., and Hochberg, Yosef, *Multiple Comparisons and Multiple Tests Using the SAS System*, Cary, NC: SAS Institute Inc., 1999, 416pp.