

Writing up the quadratic regression analysis for nitrate removal
Burchell and Osborne, March, 2003

A multiple regression model which takes the mean nitrate removal (mnr) to be quadratic in time (t) was fit using least squares:

$$mnr_i = (\beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})t_i + (\beta_5 + \beta_6 x_{i2} + \beta_7 x_{i3} + \beta_8 x_{i4})t_i^2.$$

Here $i = 1, \dots, n$ indexes the entire sample of n observations and x_{i2}, x_{i3} and x_{i4} are indicator variables for the last three treatments. For example,

$$x_{i2} = \begin{cases} 1 & \text{observation } i \text{ from treatment 2} \\ 0 & \text{else} \end{cases}$$

Defining the (full) model this way constrains the means to have the same zero intercept, but allows linear and quadratic coefficients to vary by treatment. Hypotheses of treatment effects can be written using linear constraints on the coefficients in the multiple regression. For example, a test for the reduced model involving no treatment effects at all can be written with the following 6 constraints:

$$H_0 : \beta_2 = \beta_3 = \beta_4 = \beta_6 = \beta_7 = \beta_8 = 0.$$

To test for the equality of a pair of treatments, take treatments 2 and 3 for instance, while allowing the other two means to vary freely, the reduced model can be written with the two constraints

$$H'_0 : \beta_2 = \beta_3 \text{ and } \beta_6 = \beta_7.$$

Tests comparing the reduced, or nested models with the full model can be constructed using appropriate F -ratios. To test H_0 above, use the statistic

$$F = \frac{(MSE(H_0) - MSE(\text{full}))/df}{MSE(\text{full})}$$

where df is the number of linear restrictions placed on the full model needed to get the reduced model of interest. The degrees of freedom associated with this F -ratio under H_0 are df in the numerator and $n - 8$ in the denominator. Similarly, an F -test for H'_0 is based on the statistic

$$F = \frac{(MSE(H'_0) - MSE(\text{full}))/df}{MSE(\text{full})}$$

on 2 and $n - 8$ degrees of freedom.

Residual plots did not indicate any obvious lack-of-fit or inhomogeneity of variance.