Additional Problems for HW13, Fall 2009

1. Let $X \sim \Gamma(n, \beta)$.

   (a) Show that
   $$\lim_{n\to\infty} Pr(\frac{X - n\beta}{\sqrt{n\beta^2}} \le t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du.$$

   (b) Show that $\sqrt{2X} - \sqrt{2n\beta} \to_d N(0, \beta/2)$ as $n \to \infty$.

2. Let $X_1, \cdots, X_n$ be n independent observations from the density
   $$f(x) = \frac{1}{2}x^2 e^{-x}, \quad \text{for } \ x > 0.$$

   Let $T_n = \frac{n}{\sum_{i=1}^{n} X_i^{-1}}$.

   (a) Find the constant θ such that $T \to^p \theta$.

   (b) Is it true that $T \to^{\text{a.s}} \theta$? Justify your answer.

   (c) Find constants a and b such that $\sqrt{n}(T_n - a)/b$ converges in distribution to $N(0, 1)$ as $n \to \infty$.