Show ALL your work, along with JUSTIFICATION for the steps you take.

1. (25 points) The times that a cashier spends processing each person’s orders are independent random variables with mean 2.5 minutes and standard deviation 2 minutes. What is the approximate probability that it will take more than 4 hours to process the orders of 100 people?
2. (35 points) Suppose $Y_1, Y_2, \ldots, Y_n$ forms a random sample from an $Exp(\beta)$ distribution.

(a) Find the cumulative distribution function (c.d.f) of $U = Y_{(1)}$.

(b) Use part (a) to show that the probability density function (p.d.f) of $U$ is one of our “familiar forms.”

(c) What is the expected value of $U$?
3. (40 points) Suppose $Y_1, Y_2, \ldots, Y_n$ forms a random sample from a $\chi^2_1$ distribution.

(a) **Derive** (do not simply quote a result from class) the exact distribution of $U = \sum_{i=1}^{n} Y_i$.

(b) Assuming $n$ is large, use the Central Limit Theorem to approximate the distribution of $U$.

(c) Using the results of parts (a) and (b), can you suggest a “normal approximation to the chi-square distribution”? 
Bonus (5 points): Attempt the following only after you are satisfied with all other responses. Partial credit will not be given.

Suppose $Y_1, Y_2, Y_3, Y_4$ forms a random sample from a standard normal distribution. What is the distribution of

$$\frac{Y_1 + Y_2}{\sqrt{Y_3^2 + Y_4^2}}$$

You make quote a theorem or two; there is no need to derive the distribution “from scratch.”