Show ALL your work, along with JUSTIFICATION for the steps you take.

1. (55 points) Suppose the random variables $Y_1$ and $Y_2$ have joint p.d.f.

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} \frac{1}{8} y_1 e^{-(y_1 + y_2)/2} & 0 < y_1, y_2 \\ 0 & \text{otherwise}. \end{cases}$$

(a) Find the variance of $5Y_1 - Y_2$.
(b) Let $U_1 = Y_2 / Y_1$ and $U_2 = Y_1 + Y_2$. Find the joint p.d.f. of $U_1$ and $U_2$.
(c) Refer to part (b). Are $U_1$ and $U_2$ independent?
Use this page as you wish.
2. (25 points) Assume that $Y_1, Y_2, Y_3, Y_4$ are random variables with

\[
E(Y_1) = 1 \quad V(Y_1) = 1 \quad \text{Cov}(Y_1, Y_2) = 1 \quad \text{Cov}(Y_2, Y_4) = 0
\]
\[
E(Y_2) = 3 \quad V(Y_2) = 9 \quad \text{Cov}(Y_1, Y_3) = -2 \quad \text{Cov}(Y_3, Y_4) = 0
\]
\[
E(Y_3) = 2 \quad V(Y_3) = 4 \quad \text{Cov}(Y_1, Y_4) = 0
\]
\[
E(Y_4) = 0 \quad V(Y_4) = 1 \quad \text{Cov}(Y_2, Y_3) = -1
\]

Let $U = 7Y_1 - Y_2 + Y_3 + 2Y_4$.

(a) Find $E(U)$ and $V(U)$.

(b) Is $Y_4$ independent of $Y_1, Y_2, Y_3$?
3. (20 points) Let $Y \sim Beta(\alpha, \beta)$. Find the distribution (that is, identify by name and parameters) of $U = 1 - Y$. 
Bonus (5 points): Attempt the following only after you are satisfied with all other responses. Partial credit will not be given.
Refer to question 1. The expected value of $1/Y_1$ is $1/2$ (no need to show this!). Use this or any other information available to you to find the expected value of $Y_2/Y_1$. 