Show ALL your work, along with JUSTIFICATION for the steps you take.

1. (10 points) Suppose $Y_1 \sim Bin(10, .5)$, $Y_2 \sim Bin(10, .2)$, and $Y_1, Y_2$ are independent. Then $Y_1 + Y_2 \sim ____$. Choose one of the following, and give a reason for your answer:
   A. $Bin(20, .7)$       B. $Bin(10, .7)$       C. $Bin(10, .5)$       D. $Bin(10, .2)$
   E. None of the above

2. (10 points) Suppose $Y \sim N(\mu, \sigma^2)$. Then $\frac{Y - \mu}{\sigma^2} \sim ____$. Choose one of the following, and give a reason for your answer:
   A. $N(\mu, \sigma^2/n)$       B. $\chi^2$       C. $\chi^2_{n-1}$       D. $N(\mu^2, 1)$
   E. None of the above

3. (10 points) Suppose $Y_1, Y_2, \ldots, Y_n \overset{iid}{\sim} Exponential(2)$. Then $\sum_{i=1}^n Y_i \sim ____$. Choose one of the following, and give a reason for your answer:
   A. $Exponential(2n)$       B. $Exponential(2)$       C. $Gamma(1, 2)$       D. $Gamma(n, 2)$
   E. None of the above
4. (20 points) Suppose $Y_i \sim \text{Poisson}(\lambda_i)$, $i = 1, 2, \ldots, n$, and $Y_1, \ldots, Y_n$ are independent. Use either the c.d.f., transformation, or m.g.f. technique (that is, do not simply quote a result!) to find the distribution (that is, identify by name and parameters) of $U = \sum_{i=1}^{n} Y_i$. 
5. (20 points) Let \( Y \sim Beta(2,1) \). Find the distribution (that is, identify by name and parameters) of \( U = 1 - Y \).
6. (30 points) Suppose $Y_1 \sim Uniform(0, 4)$, $Y_2 \sim Uniform(3, 5)$, and $Y_1, Y_2$ are independent. Find the joint probability density function of $U_1 = Y_2 - Y_1$ and $U_2 = Y_2$. 
Bonus (5 points): Attempt the following only after you are satisfied with all other responses. Partial credit will not be given.

Suppose random variables $Y_1, Y_2, Y_3$ are mutually independent. Fill in distributions for these random variables so that $Y_1 + Y_2 + Y_3 \sim Gamma(13, 3)$:

$$\begin{align*}
\text{independent} & \quad \left\{ 
\begin{array}{c}
Y_1 \sim \_\
Y_2 \sim \_\
Y_3 \sim \_
\end{array}
\right.
\end{align*}$$