Show ALL your work, along with JUSTIFICATION for the steps you take.

1. Short answer questions (that still require justifications).
   (a) (10 points) Suppose $Y \sim F_{8,20}$. Find $P(Y > 0.25)$.

   (b) (15 points) Suppose $Z_1, Z_2, Z_3 \sim Normal(0,1)$. Find the distribution of $\frac{Z_1 - Z_2}{\sqrt{2Z_3}}$.

2. (25 points) Suppose $Y_1, Y_2, \ldots, Y_{50}$ forms a random sample from a $Uniform(0, \theta)$ distribution. Make appropriate comments on the (approximate?) distributions of $\overline{Y}$, $\sum_{i=1}^{50} Y_i$, and $S^2$. 
3. (25 points) Suppose $Y_1, Y_2, \ldots, Y_n \overset{iid}{\sim} \text{Normal}(0, 0.25)$. Use the moment-generating-function technique (i.e., do not simply quote a theorem!) to find the distribution of $U = 2 \sum_{i=1}^{n} Y_i$. 
4. (25 points) Suppose a system of four components is constructed in parallel as shown below. This parallel construction means that all four components are simultaneously activated and the system will fail when the **longest lasting** component fails. If the lifetimes of these components are independent and all follow the exponential distribution with mean equal one hour (i.e., $Y_1, Y_2, Y_3, Y_4 \sim \text{Exponential}(1)$, $Y_i$ = lifetime of the $i^{th}$ component), what is the probability that the system fails within two hours?
Bonus (5 points): Attempt the following only after you are satisfied with all other responses. Partial credit will not be given.

Suppose $Y_1, Y_2, \ldots, Y_{10}$ are mutually independent and $Y_i \sim \chi_i^2$. What is the distribution of $U = \sum_{i=1}^{10} Y_i$?