Show ALL your work, along with JUSTIFICATION for the steps you take.

1. (25 points) John Brown is a student whose probability of earning an A in a course remains constant at 70%, irrespective of the course and across time. John needs to take a total of 41 courses to get his degree. His parents have promised that as soon as he gets 10 A’s he can move from the dormitory into his own apartment. They have also promised that for each A he receives they will add an additional $2500 to his trust fund.

   (a) How many courses will John need to take, on average, before he can move into his own apartment?

   (b) How much money can John expect his parents to add to his trust fund based on his grades?
2. (30 points) A mattress manufacturer uses a variety of measurements to assess the quality of its mattresses. One such measurement is the weight required to cause the springs to collapse 2 inches. Much historical data has been used to determine that this weight is approximately distributed as normal with mean 200 pounds and standard deviation 10 pounds. A consumer products agency has just recently issued criteria for labeling a mattress “firm, but comfortable.” This criteria says the weight to collapse springs 2 inches should ideally be 190 pounds with limits obtained as ±12 pounds.

(a) What is the probability that the manufacturer’s mattresses will be outside of the specification limits for being labeled “firm, but comfortable?”

(b) Suppose a mattress has met the specification limits given by the agency. What is the probability that the weight will be within 2 pounds of the target?

(c) Suggest a possible set of limits that would exclude only 10% of the manufacturer’s mattresses.
3. (15 points) In the inspection of a fabric produced in continuous rolls, the occurrence of imperfections follows a Poisson process with a mean of 3 imperfections per ten yards. Find the probability that

(a) ten yards of the fabric will have 3 imperfections
(b) twenty yards of the fabric will have at most 6 imperfections
4. (10 points) Suppose \( Y \sim Bet(3, 2) \). Find \( E \left( \frac{1}{1-Y} \right) \) and show that it is not equal to \( \frac{1}{E(1-Y)} \).

5. (15 points) A shipment of 80 burglar alarms contains 4 that are defective. If 5 of these are randomly selected and shipped to a customer, find

(a) the probability that the customer will receive exactly one bad unit
(b) the number of bad units the customer can expect to receive.
6. (10 points) Suppose \( Y \sim \chi^2_k \) and \( \text{Var}(Y) = 8 \). Find \( P(7.78 < Y < 14.86) \).

Bonus (5 points): Attempt this only after you are satisfied with your responses to the other questions. Partial credit will not be given, and FULL justification of the steps of a correct response is required for credit.
Suppose \( Y \sim U(21, 35) \). Then \( P(22 < Y < 30) = P(29 < Y < 37) \). True or false? Provide justification.