Show ALL your work, along with JUSTIFICATION for the steps you take.

1. (20 points) Suppose the random variable $Y$ has probability density function (p.d.f.)
   \[ f(y) = 0.25 \text{ for } y = 1, 2, 3, 4 \text{ and zero elsewhere.} \]
   
   (a) Find and graph the cumulative distribution function (c.d.f.) of $Y$.
   
   (b) What is the median of the distribution?
   
   (c) Find the 60\textsuperscript{th} percentile of the distribution.
2. (18 points) A box contains 6 white balls and 4 black balls. Balls are removed from the box without replacement until either a white ball is removed or 3 balls have been drawn. Let $Y$ be the random variable that counts the number of balls removed from the box.

(a) Find the probability density function (p.d.f.) of $Y$.

(b) Find the expected number of balls removed from the box.
3. (20 points) Suppose the random variable $Y$ has cumulative distribution function (c.d.f)

$$F(y) = \begin{cases} 
0 & y < -1 \\
\frac{y+1}{2} & -1 \leq y < 1 \\
1 & 1 \leq y.
\end{cases}$$

Find the following:

(a) $P(-0.5 < Y \leq 0.5)$
(b) $P(Y = 0)$
(c) $P(2 < Y \leq 3)$
(d) the probability density function (p.d.f.) of $Y$. 
4. (22 points) Suppose the number of transactions handled by a bank teller in a day is a random variable $Y$ with moment generating function $m(t) = 5050t^2 + 100t + 1$.

(a) Find the mean and variance of $Y$.

(b) Find a lower bound for the probability that the teller will handle between 80 and 120 transactions in a day?

(c) Find an upper bound for the probability that the teller will handle more than 130 transactions in a day?
5. (15 points) Let \( f(y) = \frac{k}{3}, \quad -1 < y < 2 \), zero elsewhere, be the probability density function (p.d.f.) of \( Y \). Find

(a) \( k \)

(b) \( P(Y^2 \leq \frac{1}{4}) \)

(c) \( P(|Y| \leq \frac{3}{2}) \).
Bonus (5 points): Attempt this only after you are satisfied with your responses to the other questions. Partial credit will not be given, and FULL justification of the steps of a correct response is required for credit.

Let the random variable $Y$ have a probability density function (p.d.f.) $f(y)$ that is positive at $y = -1, 0, 1$ and is zero elsewhere. If $f(0) = 0.5$, find $E(Y^2)$. 