1. (30 points) Let’s complicate the story of Little Red Riding Hood. Little Red Riding Hood is a little girl who wants to travel from her house to Grandma’s house. The diagram below shows the possible paths that may be taken to Grandma’s house. The circles represent the existence of wolves who may or may not be asleep when Little Red Riding Hood passes by. If a wolf is asleep, then Little Red Riding Hood will pass unharmed; but if the wolf is awake, Little Red Riding Hood will be eaten by the wolf. There are 3 different wolves who act independently of one another. The probabilities that a wolf will be asleep when Little Red Riding Hood passes by are: 0.4 for wolf #1; 0.2 for wolf #2; and 0.5 for wolf #3. What is the probability that Little Red Riding Hood makes it to Grandma’s house?
2. (20 points) The 1996 Olympic Games will be held in Atlanta, Georgia. 1000 young men will try out for the U.S. Men’s Swim Team. Of the 1000 hopefuls, 100 are from Georgia. Only 10 men will be selected for the team.

(a) Assuming the judges have no information on the ranks or the abilities of the young men (so they are all considered to have equal ability), what is the probability that 50% of the team is from Georgia?

(b) Now suppose the judges have identified three of the young men as the top three in the U.S., so that these three will definitely be selected for the team. All three young men are from outside of Georgia. The remaining hopefuls are all considered equal in ability. What is the probability that 50% of the team is from Georgia, given that the top 3 men have already been selected?
3. (35 points) Based on the personal and family medical histories of a couple who are expecting a (single) child, studies in genetics give information about the likelihood of certain events. Suppose the probability is 0.4 that the couple will have a son. Given that the child is a boy, the probability is 0.75 that he will be born a hemophiliac; if it is a girl, the probability is 0.0 that she will be born a hemophiliac. In addition, suppose the probability is 0.2 that the couple will have a hemophiliac son born feet first. Let $A$ be the event that the couple has a son; $B$ be the event that the child is a hemophiliac; and $C$ the event the child is born feet first.

(a) The description above contains four probabilities. Give the set theoretic description of the events corresponding to these probabilities. (For example, $P(A \cap B) = 0.9$.)

We would like to be able to use a Venn diagram to represent all the possibilities, as well as their probabilities. Unfortunately, the information given above is not enough. Suppose, in addition to the above, we know $P(A \cap C) = 0.25$ and $P(A \cup B \cup C) = 0.3$.

(b) Use words to describe the two events corresponding to the probabilities just given.

(c) Draw an appropriate Venn diagram that contains all the relevant probabilities.

**Bonus (5 points):** Attempt this only after you are satisfied with your responses to the other questions. Partial credit will not be given, and FULL justification of the steps of a correct response is required for credit.

Given the couple has a hemophiliac child, what is the probability they had a son who was born feet first?
4. (15 points) **Prove** that the following statement is true:

“Suppose $A$ and $B$ are disjoint (or mutually exclusive) events, each having positive (i.e., strictly greater than zero) probability. Then $A$ and $B$ are also dependent events.”