

Forward Filtering Backwards Sampling Examples for State-Space Models

Engineering Methodology Working Group
SAMSI 2007

March 30, 2007

Generated sample data from a simple projectile motion model

- ▶ First set of data was for projectile sent straight up with out air resistance

$$y(t) = y_0 + \dot{y}_0 t - \frac{g}{2} t^2 + \epsilon. \quad (1)$$

- ▶ Second set of data included air resistance

$$y(t) = y_0 + \dot{y}_0 t - \frac{g}{2} t^2 - C_D \dot{y}^2 t + \epsilon. \quad (2)$$

Both of these were used as the basis for state-space models to generate the data with noisy observations.

- ▶ Third set of data generated to mimic results of second-order system

$$y(t) = e^{-t/5} \sin(t) + \epsilon \quad (3)$$

- ▶ A univariate state-space model was fit to the first two data sets

$$x_t = fx_{t-1} + \epsilon_t \quad (4)$$

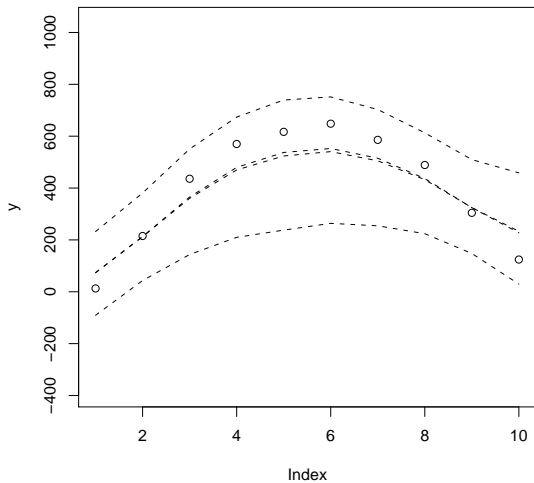
$$y_t = x_t + \eta_t \quad (5)$$

$$\epsilon_t \sim N(0, \delta_1) \quad (6)$$

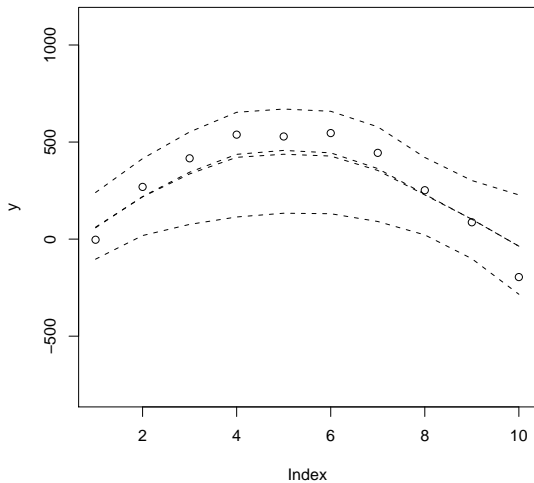
$$\eta_t \sim N(0, \delta_0) \quad (7)$$

- ▶ f , δ_1 and δ_0 are unknown, results are as follows.

Projectile Motion Without Air Resistance Univariate State



Projectile Motion With Air Resistance Univariate State



- ▶ A bivariate state-space model was fit to the first two data sets

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t \quad (8)$$

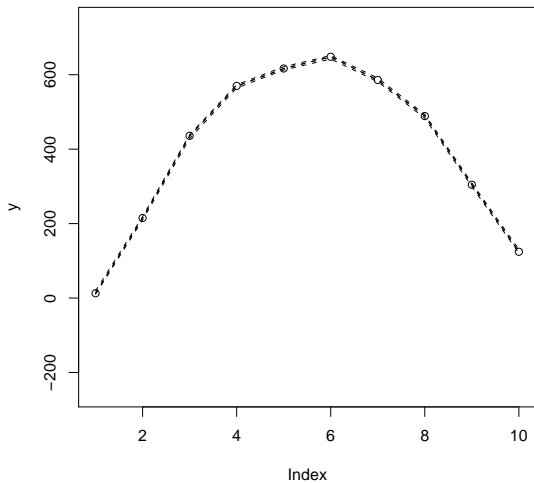
$$y_t = \mathbf{H}\mathbf{x}_t + \eta_t \quad (9)$$

$$\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \delta_1 \mathbf{I}) \quad (10)$$

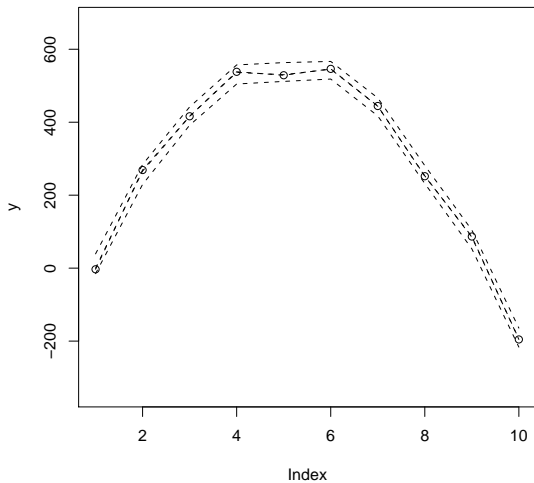
$$\eta_t \sim N(0, \delta_0) \quad (11)$$

- ▶ δ_1 and δ_0 are unknown, results are as follows.

Projectile Motion Without Air Resistance Bivariate State



Projectile Motion With Air Resistance Bivariate State



- ▶ A bivariate state-space model was fit to the third data set

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t \quad (12)$$

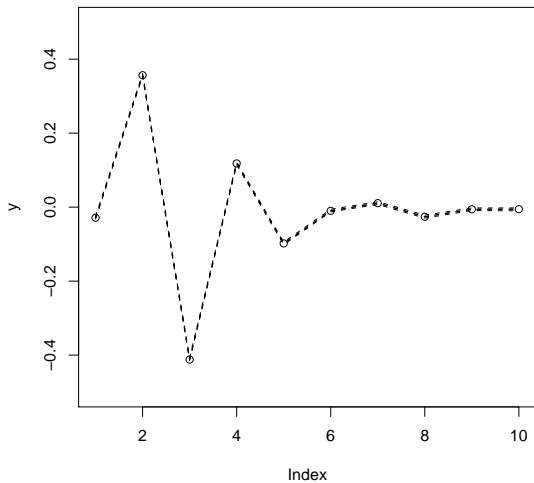
$$y_t = \mathbf{H}\mathbf{x}_t + \eta_t \quad (13)$$

$$\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \delta_1 \mathbf{I}) \quad (14)$$

$$\eta_t \sim N(0, \delta_0) \quad (15)$$

- ▶ f_{21} , f_{22} , δ_1 and δ_1 are unknown, results are as follows.

Data from Second Order System Bivariate State



- ▶ Need to include updating of x_0 and Σ_0 .
- ▶ Try some other examples i.e. estimating Δt
- ▶ Would like some measure of model performance/comparison.
- ▶ Hydrological model should be ready for April workshop.