Status Report Dynamic Modelling of Turbine Blade Vibrations

Engineering Methodology Working Group
SAMSI 2007

February 8, 2007

Lexicon

$\mathbf{x}_t = (\omega_t, \phi)$	The simulator inputs at time t including ω_t the forcing frequency and ϕ the physical system parameters.
$\mathbf{y}_t$	The observed out put of the system at time t. This is the output from the simulator.
$\boldsymbol{\xi}_t$	The system states at time t
$\mathbf{F}(\boldsymbol{\theta})$	The transition matrix containing the parameters ϕ.
$\mathbf{G}$	The input matrix describing how the known input freque… ω_t enters the emulator.
$\mathbf{H}$	The observation matrix which is known and describes which process variables are observed.
$\boldsymbol{\epsilon}_t$	The error in the emulator, possible bias, possibly the discretization error $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \mathbf{Q}_t)$.
$\boldsymbol{\eta}_t$	Observation error (?) $\boldsymbol{\eta} \sim N(\mathbf{0}, \mathbf{R}_t)$

The simulator is written in generic terms as

$$\mathbf{y}_t = f(\mathbf{x}_t)$$

The emmulator is then written as

$$\begin{aligned} \boldsymbol{\xi}_t &= \mathbf{F}(\boldsymbol{\theta})\boldsymbol{\xi}_{t-1} + \mathbf{G}\omega_t + \boldsymbol{\epsilon}_t \\ \mathbf{y}_t &= \mathbf{H}\boldsymbol{\xi}_t + \boldsymbol{\eta}_t \end{aligned}$$

Status

We assume that Rolls Royce has a model

$$\mathbf{y}_t = f(\mathbf{x}_t)$$

That is complex and expensive.

We want to construct an emulator based on a state-space model

$$\begin{aligned} \boldsymbol{\xi}_t &= \mathbf{F}(\boldsymbol{\theta})\boldsymbol{\xi}_{t-1} + \mathbf{G}\omega_t + \boldsymbol{\epsilon}_t \\ \mathbf{y}_t &= \mathbf{H}\boldsymbol{\xi}_t + \boldsymbol{\eta}_t \end{aligned}$$

We consider for now that $\mathbf{G}$ and $\mathbf{H}$ are known. The matrices $\mathbf{F}(\boldsymbol{\theta})$, $\mathbf{Q}_t$ and $\mathbf{R}_t$ are unknown and need to be estimated.
The relationship between $\boldsymbol{\theta}$ and ϕ is not known.

The solution to this problem requires two steps

1. Find the appropriate $\mathbf{F}(\theta)$, $\mathbf{Q}_t$ and $\mathbf{R}_t$ in order to construct a good emulator.
2. Define the relationship between θ and ϕ in order to allow the emulator to be used to predict new behavior for the system given new $\mathbf{x}_t$.

Part 1 can be achieved using forward filtering backwards sampling. This is essentially using the Kalman Filter inside a Gibbs sampler, the details of thsi still need to be worked out by Peter and myself the reference for this is Fr uhwirth and Schnatter (1994). The Kalman Filter is a Gaussian Process, so this method should have good emulator characteristics.

Part 2 is the more difficult portion and may require a deeper understanding of the dynamic FEM model used as the simulator. There are some promising references Hung and Senturia (1999).