Galton's Data P=Parents' average ht. S=Son's ht.

$$\begin{pmatrix} P \\ S \end{pmatrix} \sim N\left(\begin{pmatrix} 68.3 \\ 68.1 \end{pmatrix}, \begin{pmatrix} 3.19 & 2.06 \\ 2.06 & 6.34 \end{pmatrix} \right)$$

1. What is the distribution of (P-S)?
   Type of distribution
   Mean _____ Variance _____

P-S is normally distributed with mean 0.2 and variance

$$(1\ -1)\begin{pmatrix} 3.19 & 2.06 \\ 2.06 & 6.34 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 9.53-4.12 = 5.41$$

2. What proportion of sons' heights exceed their parents' average height by 1"? Pr{S-P>1} = Pr{Z>___} = 0.3037

2. $\Pr\{P-S<-1\} = \Pr\{Z< \frac{-1-0.2}{\sqrt{5.41}}\} = 0.3037$

3. Challenge: Why do you think the S variance is twice that of P? Any theory?

3. Suppose Variance{dad} = Variance{mom} = Variance{son} and mom, dad independent.

$$\begin{pmatrix} M \\ D \end{pmatrix} \sim N\left(\begin{pmatrix} * \\ * \end{pmatrix}, \begin{pmatrix} 6.34 & 0 \\ 0 & 6.34 \end{pmatrix} \right)$$

$$P = (0.5\ \ 0.5)\begin{pmatrix} M \\ D \end{pmatrix}$$

$$\sim N\left(*, \quad (0.5\ \ 0.5)\begin{pmatrix} 6.34 & 0 \\ 0 & 6.34 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \right)$$

$N(*, \quad 2(6.34/4))$

4. For later: what is the correlation between P and S?

$2.06/\sqrt{(6.34)(3.19)}$