Overview of ARIMA models

ARIMA(p,d,q)
AR Autoregressive – p lags
MA Moving Average – q lags
I   Integrated (d differences needed)

AR(1)

$$Y_t - \mu = \rho(Y_{t-1} - \mu) + e_t$$

$$Y_t - 50 = 0.8(Y_{t-1} - 50) + e_t$$

Observe Y_1=48,…,Y_{20}=55

$$Y_{21} - 50 = 0.8(55 - 50) + e_t$$

$$Y_{22} = 50 + 0.8(Y_{21} - 50) + e_{22} =$$

$$50 + 0.8(.8(Y_{20} - 50) + e_{21}) + e_{22} =$$

$$50 + 3.2 + 0.8e_{21} + e_{22}$$

Prediction

$$\hat{Y}_{22} = 50 + 3.2, \quad error = 0.8e_{21} + e_{22}$$

Assume e independent N(0,σ^2) : “White Noise”   Error variance = 1.64σ^2

Moving Average MA(1)

$$Y_t - \mu = e_t - \theta e_{t-1}$$

Autoregressive AR(1), ρ=1

$$Y_t - \mu = \rho(Y_{t-1} - \mu) + e_t$$

$$Y_t - 50 = 1(Y_{t-1} - 50) + e_t$$

$Y_t = Y_{t-1} + e_t$ same as $Y_t - Y_{t-1} = e_t$

"random walk"

Observe Y_1=48,…,Y_{20}=55

$$Y_{21} = 55 + e_t, \quad \hat{Y}_{21} = 55, \quad error = e_t$$

$$Y_{20+L} = 55 + e_{20+L} + ... + e_{21}, \quad \hat{Y}_{20+L} = 55$$

Error variance $L\sigma^2$

Every forecast is most recent observation!

Modification

$$Y_t - Y_{t-1} = e_t - \theta e_{t-1}$$

$$Y_t - Y_{t-1} = e_t - \theta e_{t-1}$$

$$e_t = Y_t - Y_{t-1} + \theta e_{t-1} = Y_t - Y_{t-1} + \theta(Y_{t-1} - Y_{t-2} + \theta e_{t-2}) =$$

$$Y_t - (1-\theta)Y_{t-1} - \theta(1-\theta)Y_{t-2} - \theta^2(1-\theta)Y_{t-3} \cdots$$

This is

$$Y_t = (1-\theta)Y_{t-1} + \theta(1-\theta)Y_{t-2} + \theta^2(1-\theta)Y_{t-3} \cdots + e_t$$

or

$$\hat{Y}_t = (1-\theta)Y_{t-1} + \theta(1-\theta)Y_{t-2} + \theta^2(1-\theta)Y_{t-3} \cdots$$

Notice that

$$\hat{Y}_t = (1-\theta)Y_{t-1} + \theta\hat{Y}_{t-1}$$

This is called exponential smoothing if $0<\theta<1$. $1-\theta$ is the "weight"

Seasonal exponential smoothing is (for monthly data)

$$Y_t - Y_{t-12} = e_t - \theta e_{t-12}$$

$$\hat{Y}_t = (1-\theta)Y_{t-12} + \theta(1-\theta)Y_{t-24} + \theta^2(1-\theta)Y_{t-36} \cdots$$